

Social Learning, the Countervailing Effects of Homophily, and Assortativity Patterns in Networks

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Abstract

We introduce a model in which homophily in social networks affects both the quality and diversity of the information to which people have access. Homophily provides higher-quality information about the actions that a group takes, since observing the payoffs of another person is more informative the more similar that person is to the decision maker. However, homophily can lead to observations about fewer actions if people similar to the decision maker choose a limited set of actions. This can lead to inefficiencies as well as inequalities across groups. We characterize conditions under which homophily hurts rather than helps social learning. Homophily lowers efficiency and increases inequality in sparse networks, but enhances efficiency and decreases inequality in dense enough networks. We also show that optimal (learning-maximizing) networks exhibit assortativity in payoff-determining characteristics, which results in incidental homophily on other innate characteristics, providing an explanation for some empirical patterns.

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1 Introduction

Chetty et al. (2022a) showed that “economic connectedness,” is by far the strongest predictor of economic mobility in the United States out of a broad set of types of social capital. Specifically, a lack of friendships across economic divides predicts that people growing up in poor households are significantly more likely to stay poor. Moreover, Chetty et al. (2022a) find that once such homophily is accounted for, the prominent relationship between inequality and immobility is mediated.¹ Places with lower levels of connection between rich and poor are both more unequal and have lower economic mobility. One hypothesis from such findings is that information bridging capital—friendships across groups that might have different information—leads to better social learning about advantageous opportunities, which can lead to both greater equality and mobility. In this paper, we build a model that allows us to study that and related hypotheses.

Given that people communicate most frequently with people with similar traits and backgrounds—due to homophily—they can end up only learning about a limited set of options, thus missing out on valuable opportunities. Different groups might not only end up with differing levels of education and outcomes within a generation, but this can persist across generations. Thus, it is important to understand how, when, and why homophily induces systematic differences in people’s beliefs, behaviors, and dynamic outcomes.

On the plus side, homophily improves how much people learn from friends’ experiences, since they can learn more from seeing whether someone similar to them succeeds at some task (e.g., going to a certain graduate school program), compared to seeing the outcome for someone with a different background and skills. For example, Porter and Serra (2020) show that female role models are influential in determining other female students’ major choices. Sorensen (2006) shows that employees learn more about health plan choices from colleagues with similar backgrounds, Conley and Udry (2010) show that farmers learn more about agricultural techniques when their peers have a similar wealth level, and Malmendier and Veldkamp (2022) examine the influence of who communicates information on decisions of vaccine adoption among other things (see also the seminal work by Katz and Lazarsfeld (1955)). Thus, homophily can improve the *quality* of information that someone gets from their network.

On the minus side, homophily can lead a person to learn only about a limited set of choices. If a person’s friends are from predominately one group, and they tend to make the same choice, then that person ends up learning about that choice, but not about others. For example, many investors invest disproportionately in domestic equities, ignoring the benefits of diversifying into foreign equities (French and Poterba, 1991).² Therefore, homophilistic

¹The relationship between inequality and immobility is known as the Great Gatsby Curve: see Corak (2016); Jackson (2019, 2021) for background.

²This behavior is not easily explained by direct barriers, e.g., transaction costs and capital controls (Ahearne, Grier, and Warnock, 2004). Information asymmetries and lower resulting posterior/residual risk seem to be key drivers of such biases (Ahearne, Grier, and Warnock, 2004; Van Nieuwerburg and Veldkamp, 2009; Portes and Rey, 2005).

groups of people can end up herding on inefficient decisions because those are the ones about which they have the best information, which then becomes persistent. Thus, homophily can limit the *quantity and diversity* of network information.

We model this tradeoff by having agents choose between a “safe” action with payoff normalized to 0 and a “risky” action with unknown payoff. For instance, an agent may be choosing between taking a minimum wage job or going to a university. Agents know their costs, but do not know the benefit of the risky action.³ Agents with costs above the unknown benefit would be better off taking the safe action, while all those with costs below the benefits should take the risky action.

The model has overlapping generations. Current agents see the choices of their friends from the previous cohort, know their friends’ costs, and see whether their friends succeeded or failed (got a net positive or negative payoff) if they take the risky action. Two things limit agents’ learning. One is that they may have a different cost than their friend(s): for instance an agent who has a high cost who sees a low-cost agent succeed with the risky action is not sure whether they will also succeed. The other is that they only learn about the risky action’s payoff if some of their friends take the risky action. This enables us to study these competing forces: gaining more precise information from other agents who have similar costs, but potentially failing to learn about the risky action if observed agents do not take it.

In particular, agents belong to one of two groups: blues or greens (e.g., income groups, ethnic groups, genders, caste groups, etc.). These groups can have different cost distributions. Homophily in terms of greens being more likely to have green friends, and blues more likely to have blue friends, means that friends are more likely to have similar costs than two people at random. This fuels the positive influence of homophily. However, in equilibrium blues and greens end up taking the risky action with different probabilities, which means that an agent is more or less likely to have a chance to learn about the risky action depending on which group a friend belongs to.

We find that having fewer friends or facing greater uncertainty about the payoff to the risky action increase the negative impact of homophily (inducing more learning failures for a group). In particular, with small numbers of friends and high amounts of homophily, a group can end up herding on inefficient decisions because of a lack of information about the risky action. Once people have more friends or face less uncertainty, then they are more likely to observe the risky action being taken (or make inferences from it not being taken) which improves learning of both groups, and eventually it becomes more important to be learning from people with similar costs, and then homophily is beneficial.

It also turns out that higher correlations between the costs across groups increase the positive impact of homophily. The intuition behind this is layered and nuanced. On the one hand, higher correlations in costs across groups leads to greater learning across groups,

³This is a somewhat artificial distinction, and one could add uncertainty over costs. What is important is that there is heterogeneity in overall payoffs and risky-action payoffs are uncertain to some agents at an interim stage.

which would superficially seem to lower the benefit of homophily. However, this also means that all groups learn more about the risky action, which then has a feedback of increasing the benefit of homophily. This shows how endogeneity of the action can reverse the impact of homophily.

The model also provides a first explanation for some new observations about homophily. A recent empirical investigation of homophily among university students (Jackson, Nei, Snowberg, and Yariv, 2022) identifies several new facts. First, there are more individuals who have very high and very low levels of homophily than what would happen with homophilistic random link formation (e.g., in a stochastic block model).⁴ Second, there is assortativity in this homophily: the most homophilous individuals are relatively more likely to be friends with the others who are most homophilous, and the least homophilous are more likely to be friends with the least homophilous. Third, and hardest to explain is that there is negative assortativity in cross-group friendships: cross-group friendships are more likely to be between a low-homophily type from one group and a high homophily type from another group.

Our model provides an explanation for all three facts, as we show in Section 4. In particular, we show that the optimal network structure for social learning (for all agents) is one in which there is assortativity in *costs*. We then show that if the two groups have different distributions of costs—so that for instance, greens are relatively more likely to have higher costs than blues—then having friends with similar costs induces all three observed facts about homophily when it comes to friendships in blue-green space. That is, assortativity in cost space when projected into blue-green space induces homophily patterns among blues and greens that exactly match the observed empirical patterns. High cost greens are more homophilous than low cost greens, and the reverse for blues. High cost greens are more likely to be connected to other high cost greens, and thus to the most homophilous agents. And when greens connect to blues, it is the most homophilous greens (high cost ones) with the least homophilous blues (the high cost ones).

This is a phenomenon that we term “incidental homophily.” Having homophily in one dimension (e.g., costs), and having that dimension not be independent of another dimension (e.g., blue and greens having different cost distributions), induces homophily in the blue-green space that has individual level heterogeneity in homophily, homophily in that homophily, and negative cross-type correlations.

Our paper relates to various strands of the social learning literature. The herding here differs from the usual forms of herding that we term (Banerjee, 1992; Bikhchandani, Hirshleifer, and Welch, 1992) *inference herding* and that occur due to cascading inferences of agents who end up choosing actions independently of their private information. Here the herding comes from portions of the population not observing the payoffs to the risky action,

⁴As a simple numerical illustration, suppose that each person has two friends, and those each have a p chance of being same ethnicity. Then a fraction p^2 of the population should have both friends same ethnicity and $(1-p)^2$ fraction of the population should have neither friend same ethnicity, while $2p(1-p)$ should have one friend of the same ethnicity. In the data, there are relatively higher fractions having both or neither, and lower fractions with intermediate levels of homophily.

even when it is taken by some other group. We refer to this type of herding as *sample herding*, to distinguish it from inference herding.

Our model draws on the idea that people learn more from those most similar. That idea has been explored previously in other contexts (Kets and Sandroni, 2016; Sethi and Yildiz, 2016), including work that explores the tradeoff between learning from similar versus diverse sources; e.g. Aral and Van Alstyne (2011). Our work provides new insights into how homophily leads to sample herding and its efficiency and inequality consequences.

Our social learning model is “*active*” in that information is endogenously generated depending on the choices of the agents. The effects of homophily have been previously studied in “*passive*” settings, in which people communicate exogenously given information through a network. For example, Golub and Jackson (2012) show that homophily can slow learning in a model of repeated communication of beliefs and updating of posteriors. In another study of passive learning, Lobel and Sadler (2016) find a tradeoff for homophily. In contrast to our result, in their setting homophily is less useful in dense networks and more useful in sparse networks. The key difference is the active/passive distinction: we have endogenous actions which generate different information as a function of the network, while in Lobel and Sadler (2016) network density effects the ability of agents to interpret the signals, but signals received are not changing with network density. Thus, having the actions chosen depend upon homophily provides new and different insights, especially regarding homophily-based herding and its implications for efficiency and inequality.

Recent work has shown that homophily in job-market referral networks can lead to differences in behaviors and outcomes across groups (Buhai and van der Leij, 2020; Bolte, Immorlica, and Jackson, 2020; Miller and Schmutte, 2021). This can result in inefficiencies, inequality, and immobility. Although some of the consequences are similar, the reasons are quite distinct and hence have different policy implications. In the context of the referral model, reducing homophily is unambiguously good, while here homophily’s effects are ambiguous and depend on the density of the network and the underlying cost and information setting.

2 The Model

2.1 Agents, Actions, and Payoffs

A continuum of agents, with size normalized to unity, each belong to one of two groups: “blues” or “greens,” denoted by $\{b, g\}$. The relative fractions are $\lambda_b > 0$ and $\lambda_g = 1 - \lambda_b > 0$.

Two actions are available to each agent. One is a “safe” action (e.g., working a minimum wage job) with a known value that is normalized to 0. The second action is the “risky” action (e.g., going to university). The risky action has a random value of its benefit v that takes on a finite set of values described by the probability function $\Pr(v)$. The risky action is costly. The distribution of costs are group-dependent and take on a finite set of values

described by the probability functions $\Pr_g(c), \Pr_b(c)$, respectively. The net payoff to an agent with cost c who takes the risky action is $v - c$ when the realized value is v .

To avoid ties, we assume that the value and cost supports are distinct. Thus, the ex post value of the risky action to any agent is either positive or negative.

Each agent knows their own idiosyncratic cost c of taking the risky action, but does not know the random value v . We describe how agents form expectations next.

2.2 Learning and Dynamics

Agents live in overlapping generations. In each period $t \in \{1, 2, \dots\}$, a new continuum of agents are born and each decide which action to take. Before making this decision, agents learn from their friends in the previous generation.

Groups can differ in the number of friends that they have and in the rate of homophily in their friendships. The underlying information network is described by a directed network (a directed graphon). Group $\theta \in \{b, g\}$ agents have an integer $d_\theta > 0$ number of friends from the previous generation. Each of these friends are from the same group with probability h_θ , and from the other group with the remaining probability $1 - h_\theta$. The actual distribution of how many of an agent of group θ 's friends are from their own group is then an independent binomial random variable with d_θ draws, each with probability h_θ . Each draw of a group picks an agent from that group with uniform probability. Thus, for instance, there is probability $h_\theta \Pr_\theta(c)$ that any given friend of a type θ agent is from group θ and has cost c , and probability $(1 - h_\theta) \Pr_{\theta'}(c)$ that the friend is from group $\theta' \neq \theta$ and has cost c . The friendships are independent across agents and friendships.⁵

Agents see each friend's action choice, group, cost, and whether their net payoff was positive or negative, but not the precise value v . In particular, each of an agent's d_θ observations is in the form $(o, c, \theta') \in \{-, +, \emptyset\} \times \mathbb{R} \times \{b, g\}$. The first dimension o is a summary of the action choice and outcome $o = +$ indicates that the observed agent took the risky action and got a positive payoff, $o = -$ indicates the agent took the risky action and got a negative payoff, and $o = \emptyset$ indicates that the agent took the safe action.

Agents update their beliefs about v based on their observed vector of d_θ signals, as a function of their knowledge of the equilibrium structure. The information that an agent of generation $t + 1$ needs as a basis for updating is the equilibrium fraction of each group θ agents with cost c taking the risky action in period t as a function of the realization of v . We denote this by $\alpha_t(\theta, c, v)$. It is useful to represent α_t as a vector in $[0, 1]^{(n_b+n_g) \times m}$ where n_θ and m are the cardinalities of the supports of $\Pr_\theta(c)$ and $\Pr(v)$.

An equilibrium defines a dynamic system starting from some initial conditions $\alpha_0 \in [0, 1]^{(n_b+n_g) \times m}$. There is a unique evolution of the system. We can also examine a steady-state (fixed point) in the function space $\alpha \in [0, 1]^{(n_b+n_g) \times m}$.

⁵We abstract from the measurability of i.i.d. random variables in a continuum. Given the finite numbers of groups and costs and a finite number of friends, these details are addressed by standard convergence arguments (Duffie and Sun, 2012).

An agent's posterior belief in period $t+1$, after observing a vector of signals \mathbf{s} from period t agents, is obtained by Bayes' Rule and denoted by $\mathbb{E}_\theta [v \mid \mathbf{s}, \alpha_t(\cdot, \cdot, \cdot)]$.

After observing the vector of signals \mathbf{s} , an agent from group θ with cost c takes the risky action if the posterior expectation is greater than c . Otherwise, the agent takes the safe action. We break ties in favor of the risky action, but any rule can be used with corresponding adjustments in the expressions.

The fraction of agents taking the risky action in period $t+1$ of group θ and cost c corresponds to the probability of a receiving a signal profile \mathbf{s} such that $\mathbb{E}[v \mid \mathbf{s}, \alpha_t(\cdot, \cdot, \cdot)] \geq c$. Thus, the dynamics of $\alpha_t(\theta, c, v)$ are:

$$\alpha_{t+1}(\theta, c, v) = \mathbb{P} \{ \mathbb{E}_\theta [v \mid \mathbf{s}, \alpha_t(\cdot, \cdot, \cdot)] \geq c \mid v, \alpha_t(\cdot, \cdot, v) \}. \quad (1)$$

It follows from (1) that $\alpha_{t+1}(\theta, c, v)$ is non-increasing in c .

However, monotonicity in v can fail because what people deduce about v depends on $\alpha_t(\theta, c, v)$. Before discussing that, we define steady-state behavior as it provides a key benchmark (given convergence).

A function $\alpha^* \in [0, 1]^{(n_b+n_g) \times m}$ is a *steady state* if it solves equation (1) with $\alpha_t = \alpha_{t+1} = \alpha^*$. A steady state exists. We omit the existence proof that follows from a standard fixed-point argument.

Steady states are not always unique. For example, if all agents have the same cost and the prior has $E[v] < c$, then for a realization $v > c$ there are two steady-state values: $\alpha^*(\theta, c, v) = 0$ for both θ and $\alpha^*(\theta, c, v) = 1$ for both θ s.

A simple variation on this yields an example of non-monotonicity of the fraction taking the risky choice taken as v increases. Consider three equally likely values of v : 0 , $c + \varepsilon$ and $c + 2\varepsilon$. For small ε it follows that $E[v] < c$. Then $\alpha^*(\theta, c, c + \varepsilon) = 1$ and $\alpha^*(\theta, c, c + 2\varepsilon) = 0$ for both θ s is a steady state.

When there are multiple steady-states, only some are stable: We call a steady state $\alpha^* \in [0, 1]^{(n_b+n_g) \times m}$ *stable* if there exists an $\varepsilon > 0$ such that for every $\alpha \in [0, 1]^{(n_b+n_g) \times m}$ in an ε -neighborhood of α^* , the dynamics converge to α^* .

3 The Advantages and Disadvantages of Homophily

We begin by analyzing a simplified version of the model that makes intuitions clear. Throughout this section the value of the risky action and the costs for each type each take two possible values ($m = n_g = n_b = 2$).

The value v is 1 with probability p , and 0 with probability $1 - p$. Thus, $\mathbb{E}[v] = p$.

The cost for a group $\theta \in \{b, g\}$ agent is $c_\theta > 0$ with probability π_θ and 0 with the residual probability $1 - \pi_\theta$.⁶ Thus, the risky action is either (ex post) optimal for all agents in the

⁶To fit with the assumption that costs and values are distinct, one can instead set the low cost to be negative or the lowest value of v to be slightly positive, so that some agents naturally enjoy the risky action regardless of its payoff.

society or else only for those who have 0 cost.

Agents with cost 0 always take the risky action and send a positive signal for both values of v , so provide no information. Thus, the dynamics are fully characterized by the dynamics of the green and blue agents who have positive costs. We denote the fraction of blue agents with cost c_b and green agents with c_g taking the risky action by $b_t(v) = \alpha_t(b, c_b, v)$ and $g_t(v) = \alpha_t(g, c_g, v)$, respectively.

In the two-value model, the dynamics are monotonic in the value of the risky action:

$$b_t(1) \geq b_t(0) \text{ and } g_t(1) \geq g_t(0)$$

for every $t \geq 1$. To see this, note that high-cost agents take the risky action if they observe a signal profile s_θ for which the posterior is sufficiently high ($\mathbb{E}_\theta[v|s_\theta] \geq c_\theta$). If $v = 0$, such signal profiles s_θ can only consist of 0-cost agents and positive cost agents who did not take the risky action ($(+, 0, \theta')$ and $(\emptyset, c_{\theta'}, \theta')$). Then if $v = 1$, agents observing any such s_θ would also take the risky action. But such agents also take the risky action if any of the positive cost agents observed took the risky action. Thus, any signal profile that has the same number of 0-cost, type θ high cost and type θ' high cost agents, regardless of their actions, induces the risky action when $v = 1$.

3.1 Full Homophily

Before analyzing how homophily impacts learning and behavior, it is useful to solve a benchmark case with extreme homophily, $h_b = h_g = 1$, which is effectively as if there is only one group since agents only ever see their own type.

Without loss of generality, consider the green group. If there is full homophily ($h_g = 1$) and at least one of π_g or d_g is not equal to 1, then the steady states are:⁷

- If $c_g \leq p$, then $g^*(0) = (1 - \pi_g)^{d_g}$, $g^*(1) = 1$ is the unique steady state.
- If $c_g > p$ and $\pi_g d_g \leq 1$, then $g^*(0) = 0$, $g^*(1) = 0$ is the unique steady state.
- If $c_g > p$ and $\pi_g d_g > 1$, then there are two steady states: $g^*(0) = 0$, $g^*(1) = 0$ and $g^*(0) = 0$, $g^*(1) \in (0, 1)$, with the latter being the only stable one.

This can be seen as follows. When $c_g \leq p$, the default action is the risky one and so the payoff is learned when the value is 1, and $g^*(1) = 1$. If $v = 0$, agents who see another high cost type either observe a negative payoff or the safe action taken - both of which reveal that the state is 0. Thus, the only agents who take the risky action are those who don't see another high cost agent, and follow their prior. That happens with probability $g^*(0) = (1 - \pi_g)^{d_g}$.

When $c_g > p$, high cost agents only take the risky action if they see information that causes them to update positively, which can only come from a high cost agent taking the

⁷If $\pi_g = d_g = 1$, then every $g^*(1) \in [0, 1]$ is a steady state.

action and getting a positive payoff. This implies that $g^*(0) = 0$, and also implies that $g^*(1) = 0$ is always a steady state. The possibility of $g^*(1) > 0$ requires high enough probability of observing other high cost agents. In that case, if there is some small ε of high cost of types who take the risky action, then others learn from that. Given that $\pi_g d_g > 1$, this converges upward as on average more see those. It does not converge to 1, as there are also some who do not observe any high cost types, or observe those from the previous generation who did not take the high action.

3.2 Partial Homophily

More interesting equilibria appear when there is interaction between the different groups, so we now focus on the case in which $h_\theta \in (0, 1)$.

The cases in which $c_\theta \leq p$ or $c_\theta > p$ for both groups are similar to the case above. The only difference is that cross-observations, and different π_θ s, lead to different proportions of blues and greens taking the risky actions. The equations are two dimensional variations of the previous section, and we leave them for the interested reader to work out.

The interesting case is when the two groups have different default actions. Without loss of generality take $c_g > p \geq c_b > 0$. Thus, c_g green agents have the safe action as their default action, and only change to risky action if they receive information causing them to update to a posterior belief with sufficiently high probability on $v = 1$, while cost c_b agents take the risky action as their default action. The latter fact helps learning. It also means that the equilibria are interior in all cases in the sense that there are always some agents taking each action, as there are agents who have priors who lead them to take the risky action with no information and others who do not, and there is always a chance that an agent does not see any high cost agents and so does not update.

Equilibria are complicated, however, as agents not only update when they see the payoff to another high cost agent, but also when they observe a high cost agent who does not take the risky action. Although equilibria cannot be solved for in closed form, we can deduce comparative statics.

The most interesting comparative static captures the dual nature of homophily. Greens are better off seeing the high cost-types who are most often taking the risky action, which could be either greens or blues depending on the context.

PROPOSITION 1 *In any stable steady state, $g^*(1)$ is increasing in h_g if and only if $\pi_g g^*(1) > \pi_b b^*(1)$.*

Thus, homophily either enhances or impedes learning depending on the equilibrium structure. $\pi_g g^*(1) > \pi_b b^*(1)$ implies that the green high cost agents who take the risky action are more plentiful than the corresponding blue agents, and thus connections to green agents are more informative than blue agents. Proposition 1 follows from a standard argument that dynamics are differentiable at a stable steady state and applying the implicit function theorem for comparative statics.

The proposition is not in terms of primitives, but in terms of equilibrium parameters. In order to derive the comparative statics in terms of primitive parameters, we focus on a specific case.

Throughout the rest of the section, we focus on equilibria with $b^*(1) = 1$. Thus, blues prefer to take the risky action unless they see a negative signal. This holds for any c_b below some cutoff level. We also maintain the assumption that $c_g > p$, which means that greens only take the risky action if they see some evidence of it being the high value state.

In such settings, greens never take the risky action in state $v = 0$, and only take the risky action in state $v = 1$ when seeing some high-cost agent taking the risky action. Blues all take the risky action in the state $v = 1$. When $v = 0$ blue agents take the risky action if they don't see any blue agents with high cost, as a blue agent with high cost not taking the action reveals that $v = 0$ as does one taking the action.⁸ The dynamics are thus:

$$\begin{aligned} g_t(0) &= 0, & g_t(1) &= 1 - \left(1 - [h_g \pi_g g_{t-1}(1) + (1 - h_g) \pi_b]\right)^{d_g}, \\ b_t(0) &= \left(1 - h_b \pi_b\right)^{d_b}, & b_t(1) &= 1. \end{aligned}$$

These dynamics are decoupled. Greens learn from both greens and blues, but blues are always taking the risky action in the high value state, and so that is a corner solution and not interacting with green behavior, so $g_t(1)$ depends only on $g_{t-1}(1)$. Blues only learn from blues as under $b^*(1) = 1$ they only modify their behavior in the low-value state and when seeing someone take the risky action and failing, and greens are not taking the risky action.

These equations yield intuitive comparative statics in terms of primitives. For these results we presume that $\pi_\theta \in (0, 1)$. This simplifies the statements as it rules out corner cases in which the inequalities are no longer strict, but the corner cases are easy to calculate.

PROPOSITION 2 *Consider a setting in which blues strictly prefer to take the risky action unless they see a negative signal. Then, given any $g_{t-1}(1)$:*

- $b_t(0)$ is decreasing in d_b and in $\pi_b h_b$,
- $g_t(1)$ is increasing in π_g , π_b and d_g .⁹

The same are true of the steady-state $g^(1), b^*(0)$ except that then $g^*(1)$ is increasing in h_g if and only if $\pi_g > \pi_b$ and $d_g > \bar{d} = \log_{1-\pi_b} \left[\frac{\pi_g - \pi_b}{\pi_g} \right]$.*

The statements about π_θ and d_θ follow from the facts that having more observations (higher d_θ) and more high cost types (higher π_θ) leads to greater information. The remaining statements are about the interaction of homophily. Blues never learn from greens, and so that accounts for $b_t(0)$ being decreasing in h_b .

⁸Given $b^*(1) = 1$, blues seeing high cost greens taking the safe action still prefer to take the risky action.

⁹If $g_{t-1}(1) = 0$ then g_t is independent of π_g .

The comparative static about h_g shows that homophily can be either beneficial or harmful. In particular, greens can learn both from blues and greens who take the risky action, and so what is important is the relative fraction of each who are taking the high action. That depends on the degrees, which is why we see $d_g > \bar{d}$. For high enough degree, greens get many observations and so are likely to take the high action, and so that favors learning from greens, to the extent that $\pi_g > \pi_b$. If instead $\pi_g \leq \pi_b$, then seeing blues always dominates seeing greens. In this case, green agents are not taking the risky action often, and increased homophily among greens reduces their information about the risky action leading them to herd to the safe action. This is the *sample herding* we referred to in the introduction.

Figure 1 illustrates this relationship and shows how the effect of degree is greater as homophily is increased.

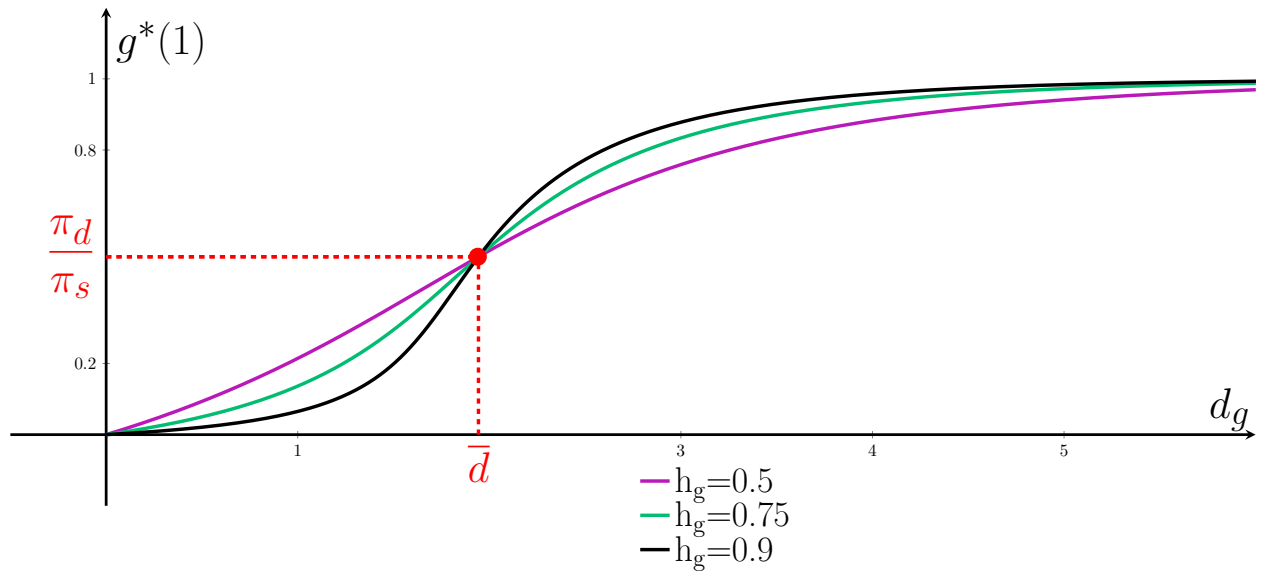


Figure 1: Steady state level of green group agents taking risky action when $\pi_s = 0.6, \pi_d = 0.3$. We show the plot for real values of d_g but the actual values of d_g are discrete.

4 Incidental Homophily and Assortativity in Costs

We now return to the general model with multiple values and costs. For the remainder of the paper, assume that for any cost in the support there is a value in the support above and below it. This ensures that the risky action should only sometimes be taken, for any cost.

Instead of just tracking homophily in blue-green types, we also allow homophily to depend on costs. Thus, we now track an agent's type as a pair $(\theta, c) \in \{b, g\} \times \mathbb{R}$. An agent with type (θ, c) has a friend of type (θ', c') with probability $h_{\theta, c}(\theta', c')$.

4.1 Optimality of Perfect Cost Homophily

If the costs of two agents are too different from each other then one learns little from the whether the other succeeds or fails at the risky action. Agents learn the most from observing others with the same cost who take the risky action, regardless of whether they are green or blue.

We say that there is *perfect cost homophily* if for each (θ, c) type, $h_{\theta,c}(\theta', c') > 0$ implies that there are no values v in the support of the risky action between c' and c .

We say that there is *complete learning* if $\alpha^*(c, \theta, v) = 1$ whenever $c < v$ and $\alpha^*(c, \theta, v) = 0$ whenever $c > v$.

Proposition 3 shows that these two conditions are equivalent.

PROPOSITION 3 *Suppose that $d_g, d_b > 1$. Complete learning is the unique stable steady state if and only if there is perfect cost homophily.*

4.2 Incidental Homophily and Assortativity Patterns

Next, we explore other implications of cost homophily. We show that homophily in one dimension (costs) can lead to incidental homophily on another dimension (blue/green), even when there is no homophily on that second dimension.

For the remainder, we take blue and greens to have the same cost supports and have values in the value support between any two costs in the cost support.

Let us say that there is perfect cost homophily with no attention to green/blue if there is perfect cost homophily and for every θ, c ,

$$h_{\theta,c}(\theta', c) = \frac{\lambda_{\theta'} \Pr_{\theta'}(c)}{\lambda_g \Pr_g(c) + (1 - \lambda_g) \Pr_b(c)}.$$

As we show next, this not only implies homophily in blue green space, but that there is heterogeneity in that homophily, and the own-type links are assortative in blue/green homophily. It also implies additional results as we state next.

We say that $\Pr_g(\cdot)$ *likelihood ratio dominates* $\Pr_b(\cdot)$ if $\frac{\Pr_g(c)}{\Pr_b(c)}$ is increasing in c on the support.

PROPOSITION 4 *If $\Pr_g(\cdot) \neq \Pr_b(\cdot)$ and there is perfect cost homophily with no attention to blue/green, then there is average homophily in blue/green: $\sum_c \Pr_{\theta}(c) h_{\theta,c}(\theta, c) > \lambda_{\theta}$.*

If additionally $\Pr_g(\cdot)$ likelihood ratio dominates $\Pr_b(\cdot)$, then homophily in blue/green is monotonic in cost: $h_{g,c}(g, c)$ is increasing in c , and $h_{b,c}(b, c)$ is decreasing in c . Moreover, there exists a threshold cost \bar{c} that determines homophily/heterophily in opposite ways across groups: $h_{g,c}(g, c) > \lambda_g$ and $h_{g,c}(g, c) < \lambda_b$ whenever $c > \bar{c}$, and the inequality is reversed if $c < \bar{c}$.

Figure 2 illustrates Proposition 4.

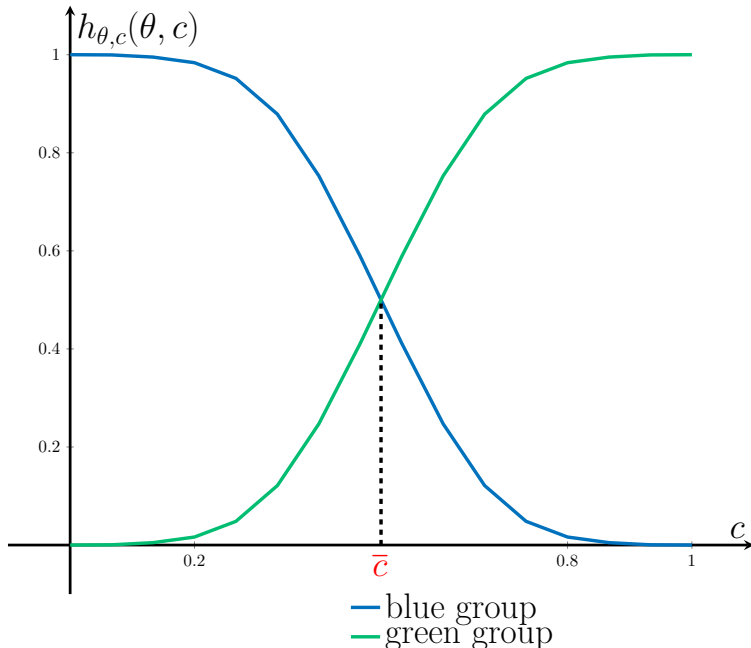


Figure 2: Blue/green homophily as a function of cost when there perfect cost homophily with no attention to green/blue and F_g likelihood ratio dominates F_b . The figure is for equal-sized groups.

Proposition 4 shows that perfect homophily in costs with no attention to green/blue leads to average homophily in greens and blues. That is, although the network is determined by costs in a color-blind way, in the resulting network the greens are more relatively more likely to be linked to greens and blues to blues. This only requires that the cost distribution differ between the two groups.

Moreover, if the groups are ordered in terms of their costs, then homophily is assortative. Higher cost agents from a higher cost group are more homophilous (on blue/green) while lower cost agents from the same group are less homophilous. Finally, green and blue agents have opposite homophily/heterophily above and below a threshold cost \bar{c} . This means that cross group links (since they are based on cost) end up being negatively assortative: homophilous greens connect with heterophilous blues and vice versa.

5 Concluding Remarks

The benefits of cost homophily only arise in the long run, approaching steady-state, and otherwise it can be inhibiting, along the lines of Proposition 2. This suggests policies that encourage cross-group (but cost-homophilistic) relationships when there are substantial differences in risky-action choices by groups (e.g., consistent with Chetty et al. (2022a)), but then encouraging pure cost-homophily once learning is occurring. More generally, policies disseminating information about success rates *by cost* can be much more enlightening than

simply providing success rates.

Finally, understanding incidental homophily seems important, given that homophily is often measured with respect to some limited set of attributes, but might be generated by others. This suggests a next step of expanding network formation models (e.g., [Currarini, Jackson, and Pin \(2009, 2010\)](#)) to understand the interaction between different characteristic dimensions ([Zuckerman, 2022](#)), and how those different attributes are distributed across the various settings in which people make friendships ([Chetty et al., 2022b](#)).

6 Proofs

Proof of Proposition 2: The results on $b_t(0)$, $g_t(1)$ and $b^*(0)$ follow directly from the dynamic equations.

We analyze $g^*(1)$, which solves

$$g = \Gamma(g) = 1 - \left(1 - [h_g \pi_g g + (1 - h_g) \pi_b]\right)^{d_g}.$$

$\Gamma(g)$ is concave, nondecreasing, and continuous, with $1 > \Gamma(1) > \Gamma(0) > 0$. Thus, a steady state exists and is unique.

Comparative statics follow from implicit function theorem. We identify the threshold \bar{d} such that $g^*(1)$ is increasing in h_g if and only if $d_g > \bar{d}$.

If $\pi_g < \pi_b$, $g^*(1)$ is decreasing in h_g for any d_g —since $\pi_g g^* < \pi_b$ for every g^* . Instead, suppose $\pi_g \geq \pi_b$. We take the natural-log of $g = \Gamma(g)$ and rearrange to obtain d_g as an increasing function of the corresponding steady state g^* :

$$d_g = \frac{\ln(1 - g^*(1))}{\ln(1 - h_g \pi_g g^*(1) - (1 - h_g) \pi_b)}.$$

Let $\bar{d} := \log_{1-\pi_b} \frac{\pi_g - \pi_b}{\pi_g}$ —the value of d_g when $g^*(1) = \frac{\pi_b}{\pi_g} \in (0, 1)$. Since d_g is increasing in $g^*(1)$, we have $d_g > \bar{d}$ if and only if $\pi_g g^*(1) > \pi_b$. Recall, $g^*(1)$ is increasing in h_g if $\pi_g g^*(1) > \pi_b$. We conclude $g^*(1)$ is increasing in h_g if $d_g > \bar{d}$. ■

Proof of Proposition 3: We first suppose there is perfect cost homophily and show the unique stable steady state is complete learning.

Consider $g_t(c, v) = b_t(c, v) = 1$ and $g_t(c', v) = b_t(c', v) = 0$ for $c' > v > c$. Next period, cost c agents observe another cost c agent taking the risky action and learn $v > c$. Similarly, cost c' agents observe another taking the safe action. This reveals $v < c'$, as otherwise observed agents would take the risky action. Thus, complete learning is a steady state.

We proceed by considering $c < v$ and $c' > v$ separately and show stability and uniqueness.

First, we show complete learning is stable for $c < v$. Observing at least one cost c agent taking the risky action fully reveals $c < v$. The fraction of cost c agents taking risky action

is lower bounded by that probability:

$$g_{t+1}(c, v) \geq 1 - (1 - h_g g_t(c, v) - (1 - h_g) b_t(c, v))^{d_g} \geq 1 - (1 - \min\{g_t(c, v), b_t(c, v)\})^{d_g} \quad (2)$$

$$b_{t+1}(c, v) \geq 1 - (1 - \min\{g_t(c, v), b_t(c, v)\})^{d_b} \quad (3)$$

Every ε -perturbation of complete learning has $\min\{b_t(c, v), g_t(c, v)\} \geq 1 - \varepsilon$. Then $g_{t+1}(c, v) \geq 1 - \varepsilon^{d_g} > 1 - \varepsilon = g_t(c, v)$ if $d_g > 1$. Thus, $g^*(c, v) = b^*(c, v) = 1$ is stable.

Second, we show if $g^*(\cdot, v)$ and $b^*(\cdot, v)$ is a stable steady state then $g^*(c, v) = b^*(c, v) = 1$ for $v > c$. Suppose not, there exists another stable steady state $g'(\cdot, v), b'(\cdot, v)$.

Steady states solve (2) and (3). The only solution different than complete learning is $g'(c, v) = b'(c, v) = 0$. However, it is not stable. Consider an ε -perturbation: $g_t(c, v) = b_t(c, v) = \sqrt{\varepsilon}$. We have $g_{t+1}(c, v) = 1 - (1 - \sqrt{\varepsilon})^{d_g} > \sqrt{\varepsilon} = g_t(c, v)$ if $d_g > 1$. Thus, every stable steady state has $g^*(c, v) = b^*(c, v) = 1$.

Third, we show that if a steady state is stable then $g(c', v) = b(c', v) = 0$. There are two possible signal profiles for cost c' agents: (i) At least one negative signal about the risky action, and (ii) No signals about the risky action. The former reveals $c' > v$. The agent can also infer $c' > v$ in the latter, since $g(c, v) = b(c, v) = 1$ whenever $v > c$ at a stable steady state.

Fourth, $g^*(c', v) = b^*(c', v) = 0$ is stable, as observing any signal fully reveals the state for every $g(\cdot, v)$ and $b(\cdot, v)$ in an ε -neighborhood of complete learning.

These four points together show that complete learning is the unique stable steady state.

For the converse, we show complete learning at the steady state implies perfect cost homophily. The proof is by contrapositive. Consider a network without perfect cost homophily. There exists type (θ, c) and (θ', c') agents such that (θ, c) agents observe (θ', c') with positive probability, and there is a value v between c' and c . We show that a steady state cannot have complete learning.

Suppose $c > v > c'$. There is a positive measure of (θ, c) agents who observe only cost c' agents taking risky action. If the posterior is above c , then there is incomplete learning. If not, for $v' > c$, there are (θ, c) agents who receive the same signal profile and have the same posterior. So, they take the safe action. This contradicts complete learning.

Suppose instead that $c' > v > c$. There are (θ, c) agents who observe only cost c' taking the safe action. If the posterior is below c , then complete learning fails. If not, for $v'' < c$ there are type (θ, c) agents who receive the same signal profile and take the safe action. This contradicts complete learning. ■

Proof of Proposition 4: We define two auxiliary vectors \mathbf{x} and \mathbf{y} : $x_c = \frac{\Pr_{\theta}(c)}{\sqrt{\lambda_{\theta} \Pr_{\theta}(c) + (1 - \lambda_{\theta}) \Pr_{\theta'}(c)}}$

and $y_c = \sqrt{\lambda_{\theta} \Pr_{\theta}(c) + (1 - \lambda_{\theta}) \Pr_{\theta'}(c)}$ for every c . Applying the Cauchy-Schwarz inequality:

$$\sum_c \left(\frac{\Pr_{\theta}(c)^2}{\lambda_{\theta} \Pr_{\theta}(c) + (1 - \lambda_{\theta}) \Pr_{\theta'}(c)} \right) \sum_c \left(\lambda_{\theta} \Pr_{\theta}(c) + (1 - \lambda_{\theta}) \Pr_{\theta'}(c) \right) \geq \left(\sum_c \Pr_{\theta}(c) \right)^2.$$

Multiplying both sides with $\frac{\lambda_\theta}{\sum_c (\lambda_\theta \Pr_\theta(c) + (1-\lambda_\theta) \Pr_{\theta'}(c))} > 0$:

$$\sum_c \Pr_\theta(c) h_{\theta,c}(\theta, c) = \lambda_\theta \sum_c \frac{\Pr_\theta(c)^2}{\lambda_\theta \Pr_\theta(c) + (1-\lambda_\theta) \Pr_{\theta'}(c)} \geq \frac{(\sum_c \Pr_\theta(c))^2}{\sum_c [\lambda_\theta \Pr_\theta(c) + (1-\lambda_\theta) \Pr_{\theta'}(c)]} \lambda_\theta = \lambda_\theta.$$

The Cauchy-Schwarz inequality holds with equality if and only if \mathbf{x} and \mathbf{y} are linearly dependent, which requires $\exists \kappa \neq 0$ such that $(1 - \lambda_\theta \kappa) \Pr_\theta(c) = \kappa (1 - \lambda_\theta) \Pr_{\theta'}(c)$ at every c . This is not possible if $\Pr_\theta(\cdot) \neq \Pr_{\theta'}(\cdot)$.

We can write $h_{\theta,c}(\theta, c) = \frac{\lambda_\theta \frac{\Pr_\theta(c)}{\Pr_{\theta'}(c)}}{\lambda_\theta \frac{\Pr_\theta(c)}{\Pr_{\theta'}(c)} + (1-\lambda_\theta)}$. It follows that $h_{\theta,c}(\theta, c)$ is increasing in c if $\frac{\Pr_\theta(c)}{\Pr_{\theta'}(c)}$ is increasing in c .

If $\frac{\Pr_\theta(c')}{\Pr_{\theta'}(c')} > 1$ then $h_{\theta,c'}(\theta, c') > \lambda_\theta$ and $\lambda_{\theta'} > h_{\theta',c'}(\theta', c')$. Let \bar{c} be the smallest such c' . If $\frac{\Pr_\theta(c)}{\Pr_{\theta'}(c)}$ is increasing then the conclusion holds whenever $c > \bar{c}$. The inequalities are reversed if $c < \bar{c}$. ■

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