Efficient Cheap Talk in Complex Environments

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Expertise in Practice

- In practice, experts often have power over decision makers.
  - Division managers over the headquarters (Milgrom and Roberts, 1988),
  - Realtors over homeowners (Levitt and Syverson, 2008),
  - OBGYNs over patients (Gruber and Owings, 1996),
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- “The power position of an expert is always overtowering.” – Weber (1922)

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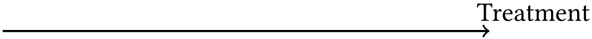
- Mismatch between models and practice – Why?

Expertise in Models v. Practice



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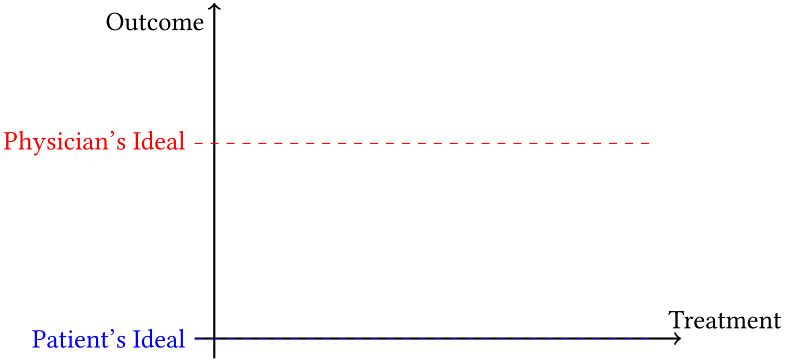

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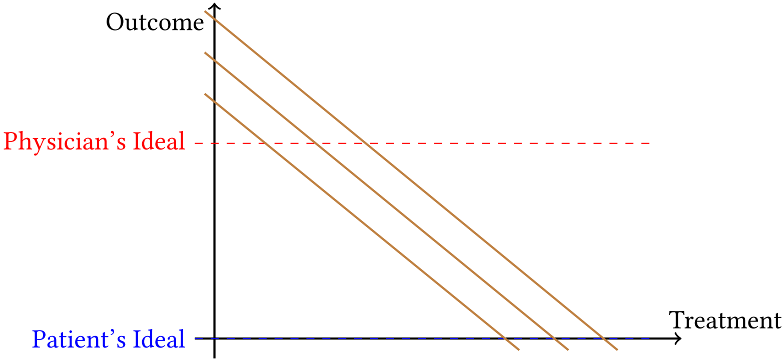

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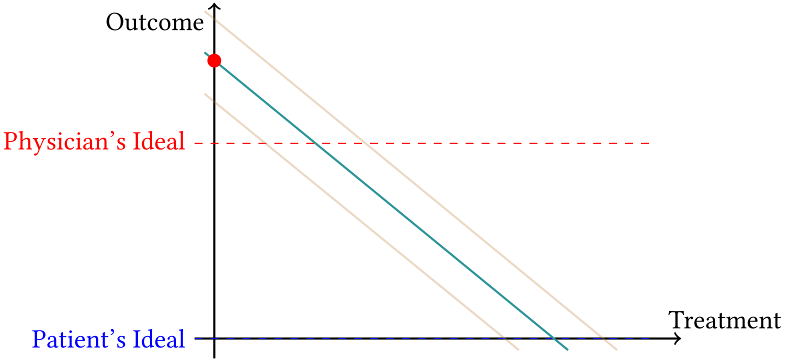

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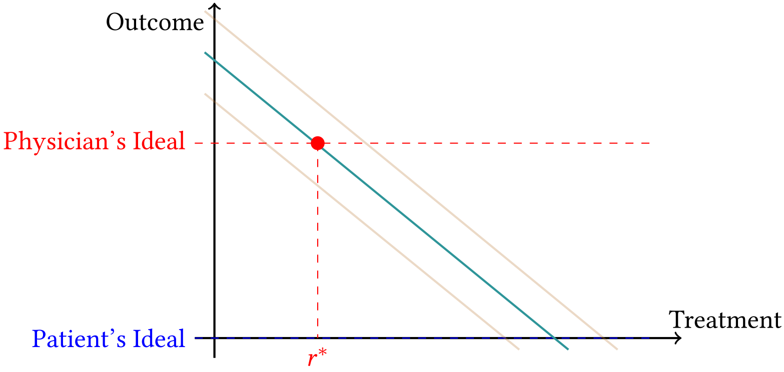

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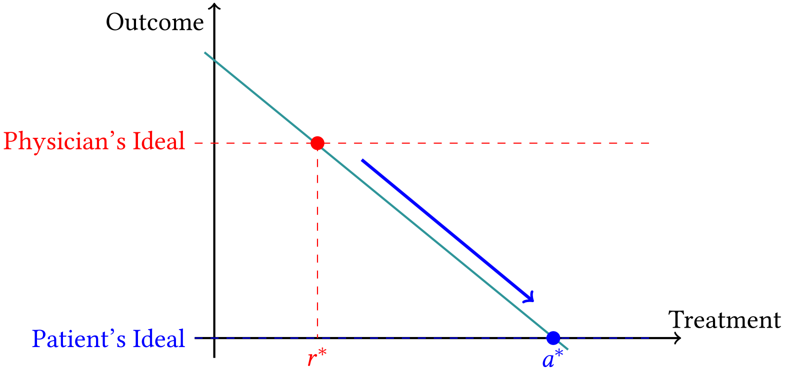

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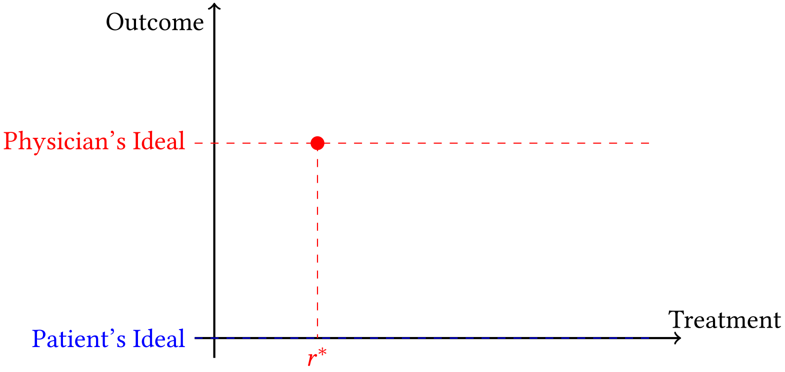

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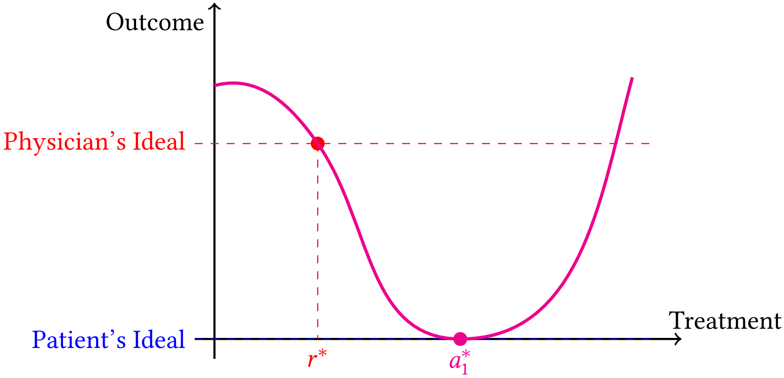

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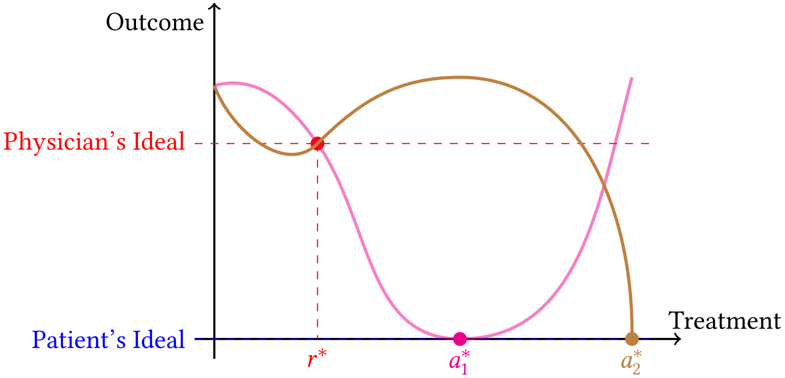

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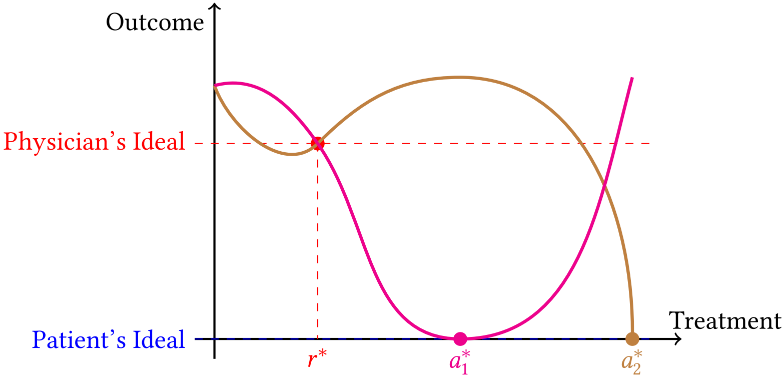

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  - Physician reveals ideal treatment → Patient inverts the mapping and learns own ideal treatment.
- **Practice:** Relationship between treatments and outcomes is highly unknown and complex.
  - Physician reveals ideal treatment → Patient cannot invert the mapping and faces uncertainty.

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- Strategic communication in complex environments where the relationship is non-invertible.
- We model the mappings from actions to outcomes as paths of Brownian Motion.
  - The expert knows the outcome of every action.
  - The decision maker knows the joint distribution of outcomes for each action.
- Equilibrium reverses the predictions of C-S:
  1. Communication is Pareto efficient.
  2. The expert has full power — the equilibrium outcome is equivalent to full delegation.

Reconciling Theory with Practice

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- Large information gaps are the source of expert power. (Weber, 1922; French and Raven, 1959).
- We show how large of an "information inequality" supports expert power by itself.

Brownian Motion and Beyond

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  - Parameterize how much is learned from the expert's ideal action.
  - Show how much learning is too much learning for supporting expert power.
- Brownian Motion, and its special features, are not necessary for expert power.
- We provide examples of other environments and extract the essential ingredient for expert power.
- A large 'information inequality' is the essential ingredient for expert power.
  - Expert can reveal her most-preferred action without eliminating all uncertainty.

1. Brownian Motion Model.
2. Results
   - Decision making without the Expert.
   - Main Result: Decision making with the Expert.
3. Extensions: Brownian Motion and Beyond.

The Model

- **Players:** Sender (the expert) and receiver (the decision maker).
- **Actions and Messages:** $\mathcal{A} = [0, q]$ for $q \in \mathbb{R}$ and $r \in \mathcal{M}$.
- **Outcomes:** $\psi : \mathcal{A} \to \mathbb{R}$ maps actions to outcomes.
- **Preferences:** $u^{S}(a) = -(\psi(a) - b)^2$ and $u^{R}(a) = -\psi(a)^2$.
  - Results hold for weakly concave $u^{R}(\cdot)$ and any $u^{S}(\cdot)$ maximized at b.

Timing of the Game

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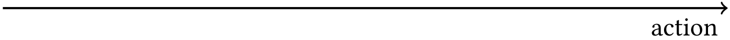
- **Solution Concept:** Perfect Bayesian Equilibrium.

Formal Definitions

Complex Environments in A Picture



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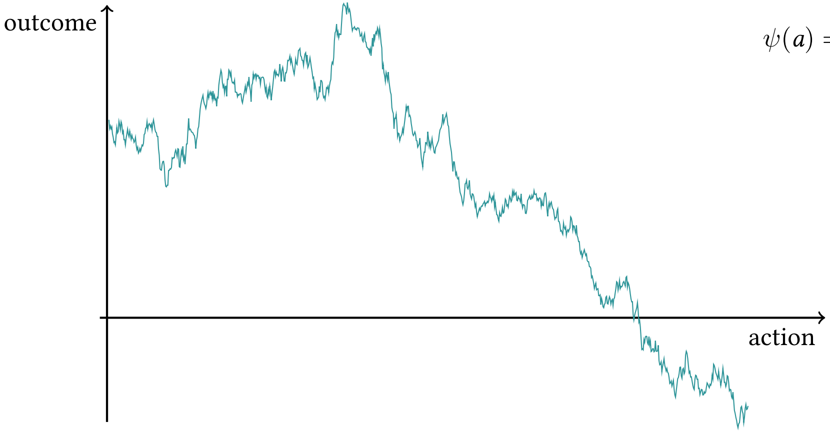

$$\psi(a) = \psi_0 + \mu a + \sigma W(a)$$

- The mapping $\psi : \mathcal{A} \to \mathbb{R}$ is given by the path of a Brownian Motion.

Details of Complex Environments

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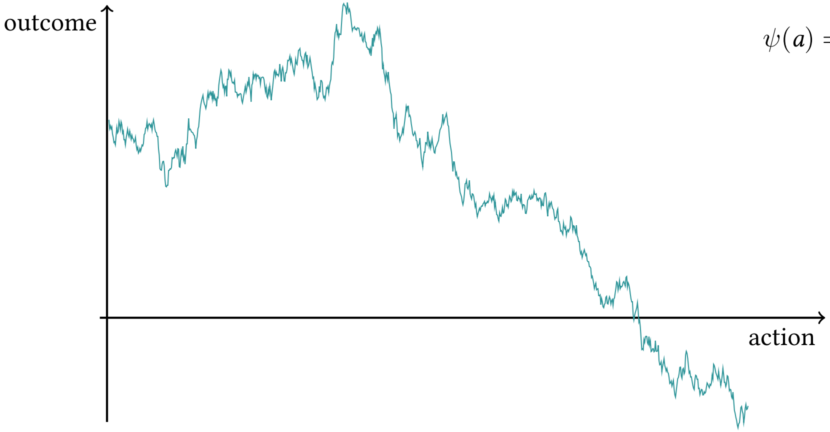

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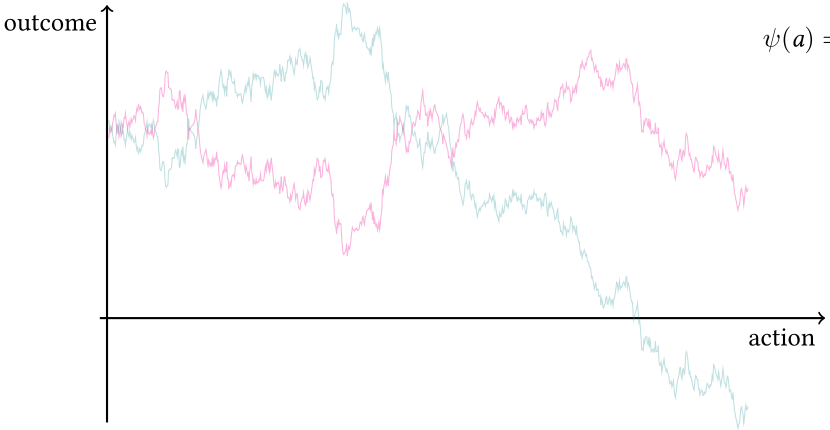

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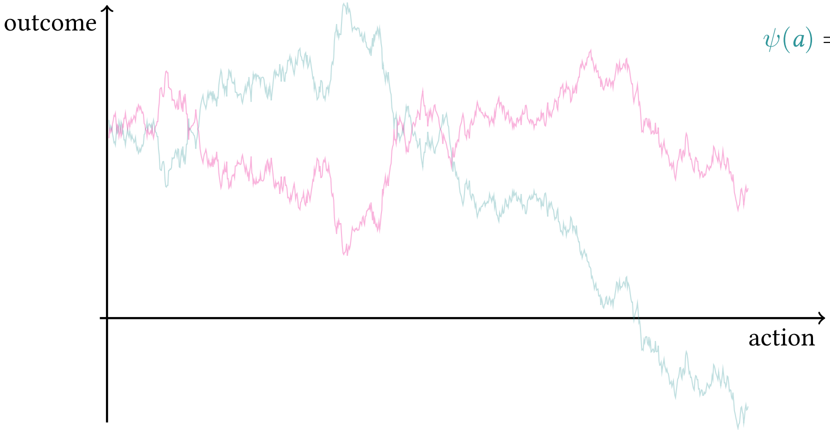
Details of Complex Environments

- Expert knows the realized mapping. Receiver does not know the realized mapping.

Receiver's Beliefs



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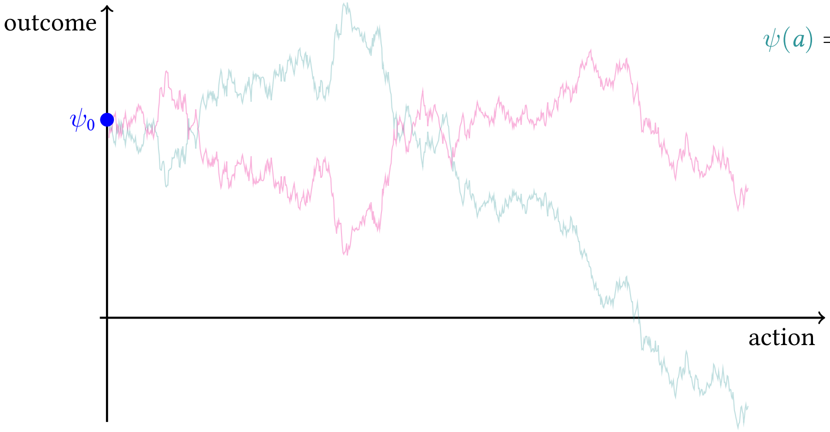

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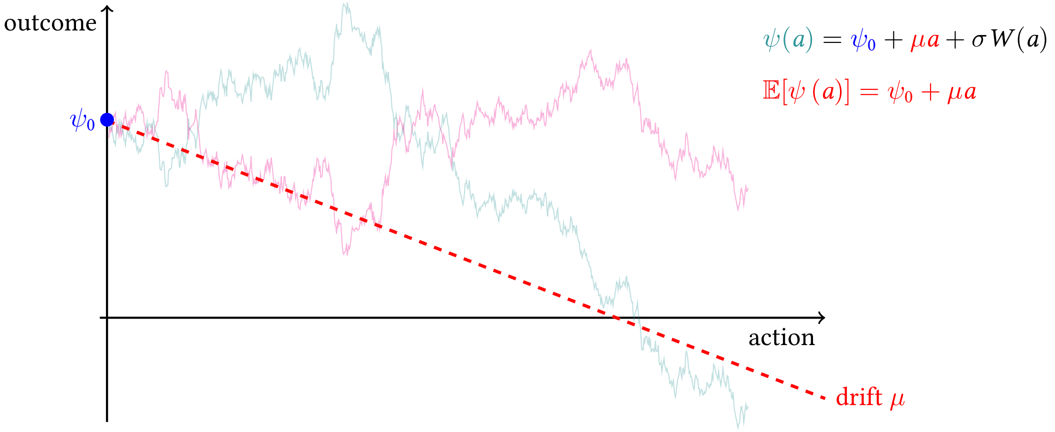

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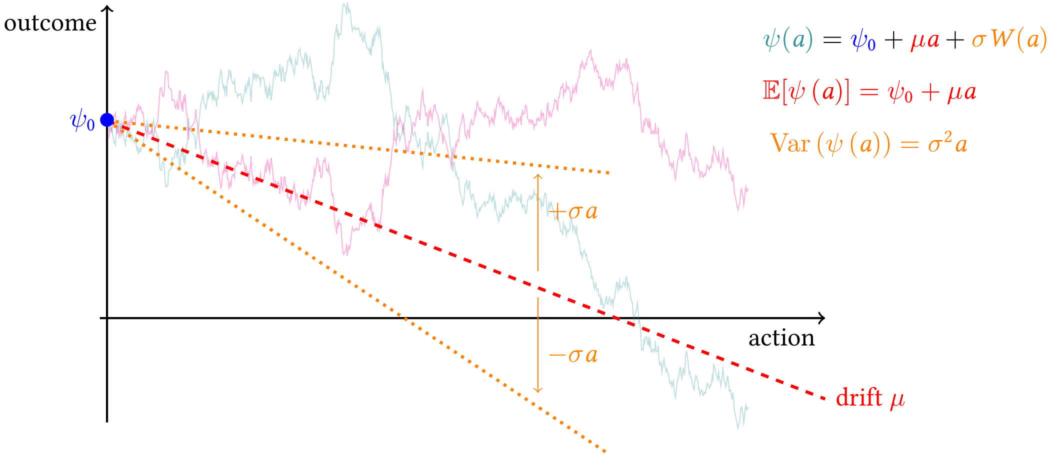

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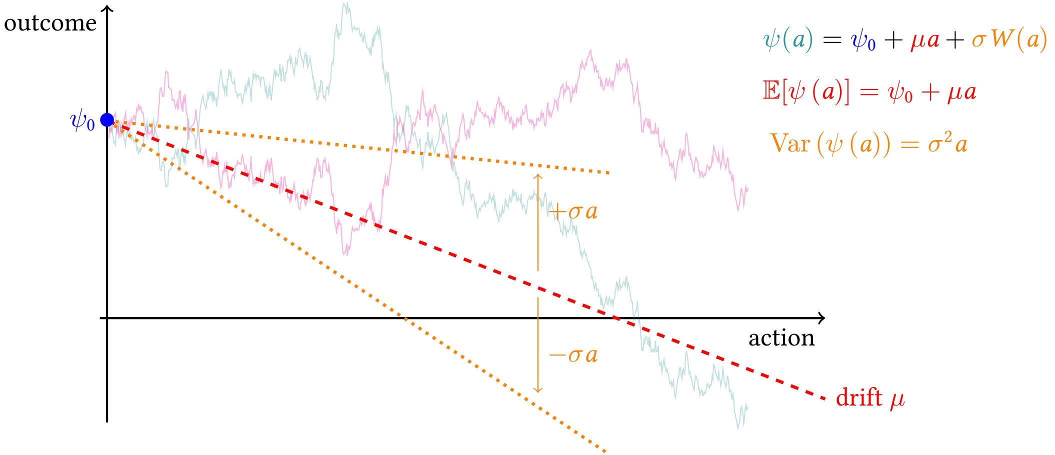

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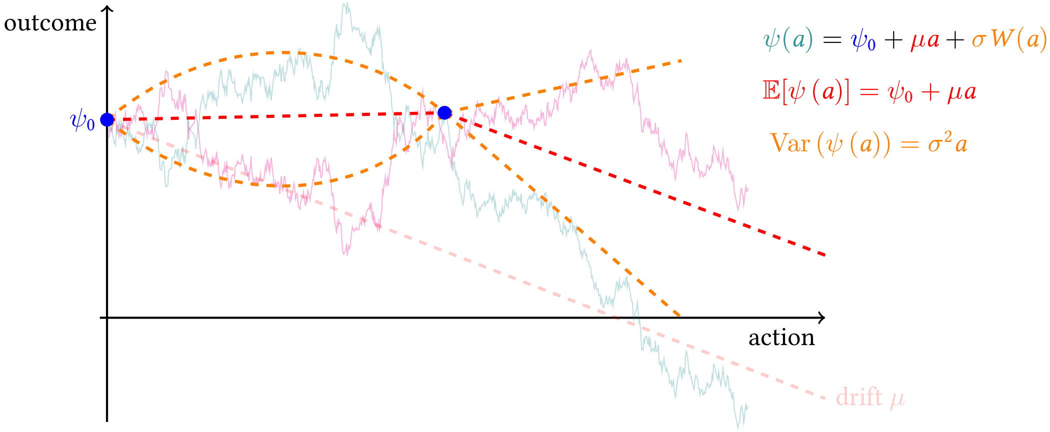

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- Learning one point in the mapping $\neq$ Learning the whole mapping.

Details for Beliefs

Simple v. Complex Environments

- **Key Difference:** How much the receiver learns from the expert's most preferred action.
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$$\psi(a) = b \Rightarrow \theta = b + a \Rightarrow \psi(a') = (b + a) - a'$$

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- **Complex Environments:** Relationship is unknown and outcomes are imperfectly correlated.
  - $\psi(\cdot)$ is a Brownian Motion with parameters μ and σ:

$$\psi(a) = b \Rightarrow \psi(a + x) \sim b + \mathcal{N}(\mu x, \sigma^2 x)$$

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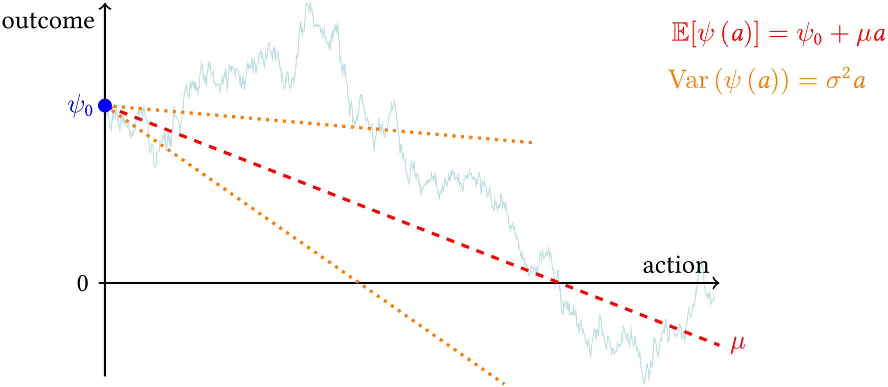
Decision Making Without the Expert



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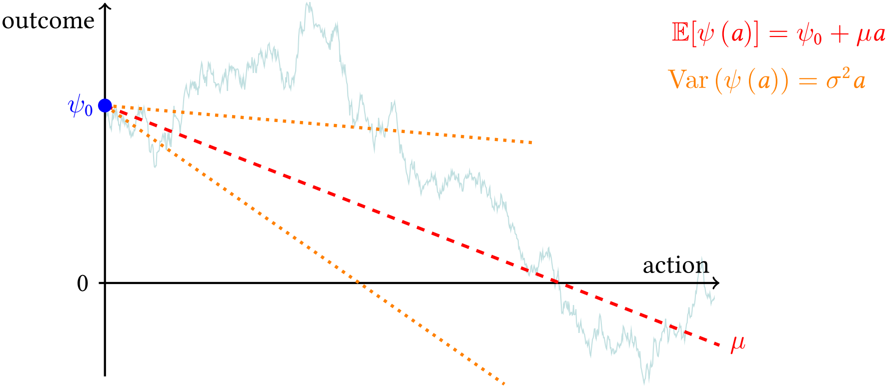

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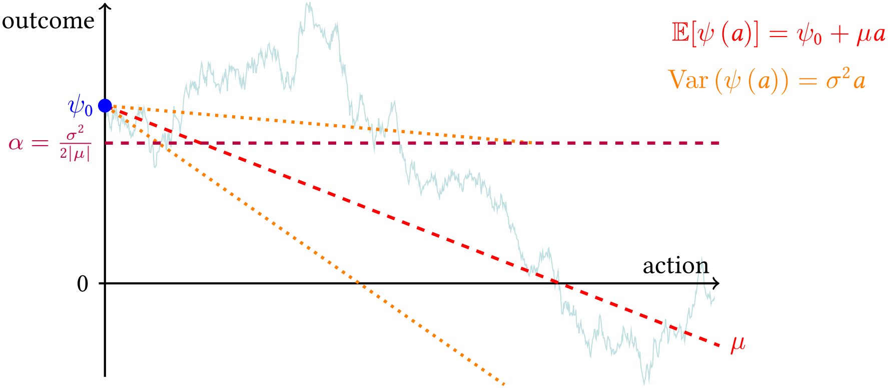

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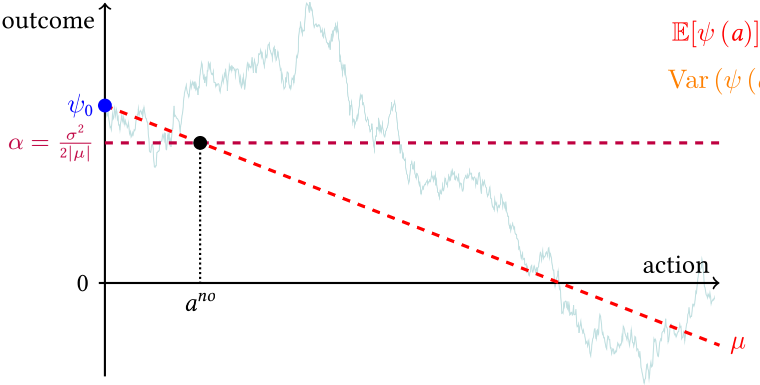
- $a > 0$ improves the outcome by μa (up until 0), but increases variance by $\sigma^2 a$.
- We call half of this ratio $\alpha = \frac{\sigma^2}{2|\mu|}$ as the **risk complexity** of the environment.

Decision Making Without the Expert



Lemma 1: Without additional information, receiver picks a^{no} based on the risk complexity α and the status-quo outcome ψ_0:

(i) If $\alpha < \psi_0$ receiver chooses a^{no} such that $\mathbb{E}[\psi(a^{no})] = \alpha$,

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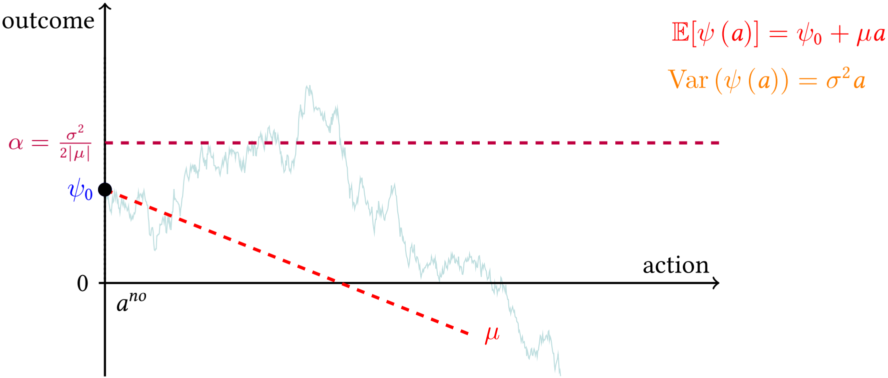

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(ii) If $\alpha > \psi_0$ then the receiver chooses $a^{no} = 0$.

Details of the proof

1. The Model: Introducing Complex Environments.
2. Results
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3. Extensions: Brownian Motion and Beyond.

Decision Making with the Expert

- We introduce the sender (expert) back into the game.
  - The sender observes the realized outcomes $\psi(\cdot)$ and recommends an action $r \in \mathcal{A}$.
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  - The sender observes the realized outcomes $\psi(\cdot)$ and recommends an action $r \in \mathcal{A}$.
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- How much power does the sender have over the final decision?
  - Full power if she can reveal her ideal action while keeping the receiver uncertain enough.

The Sender's Communication Strategy

- **First-point strategy:** Sender recommends the first of her optimal actions.

$$m^*(\psi) := \min_{a\in[0,q]} \left\{ a : a \in \arg\max_{a'\in\mathcal{A}} u^S(a' \mid \psi) \right\}.$$

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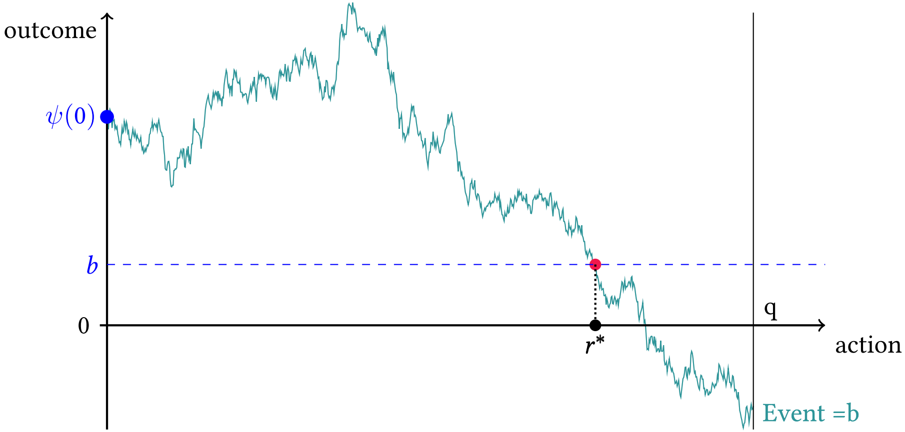
  - What does the the receiver learn from the recommendation about the path?

The Receiver's Inference Problem



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Details for Event=b

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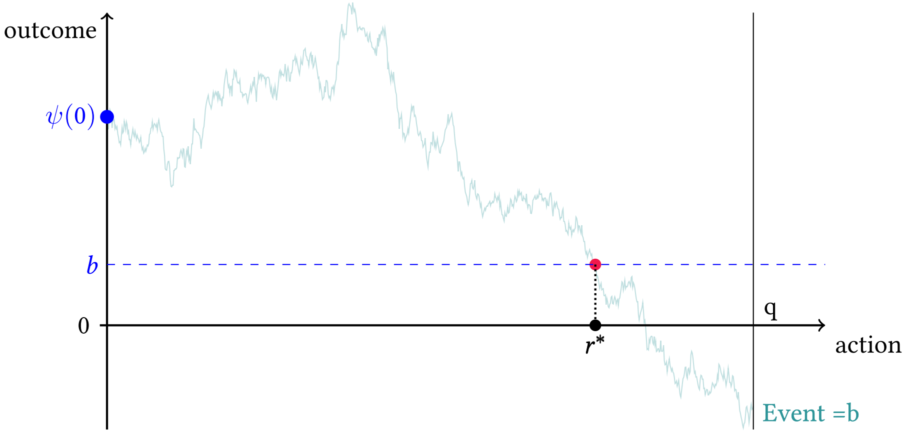

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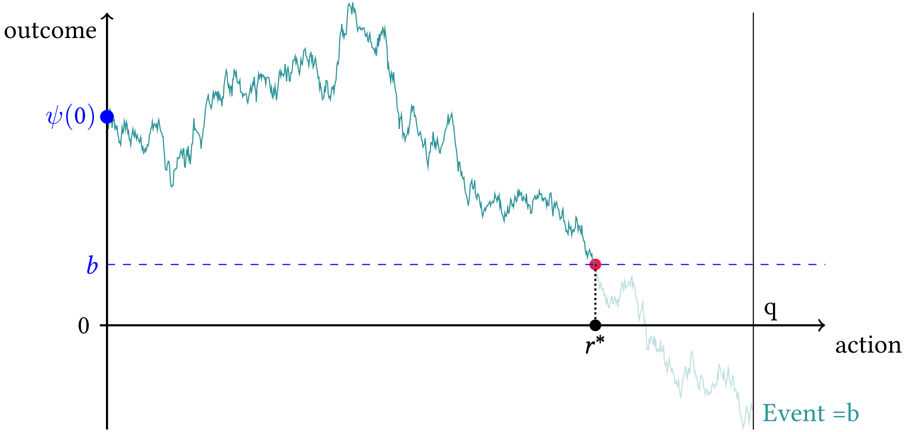

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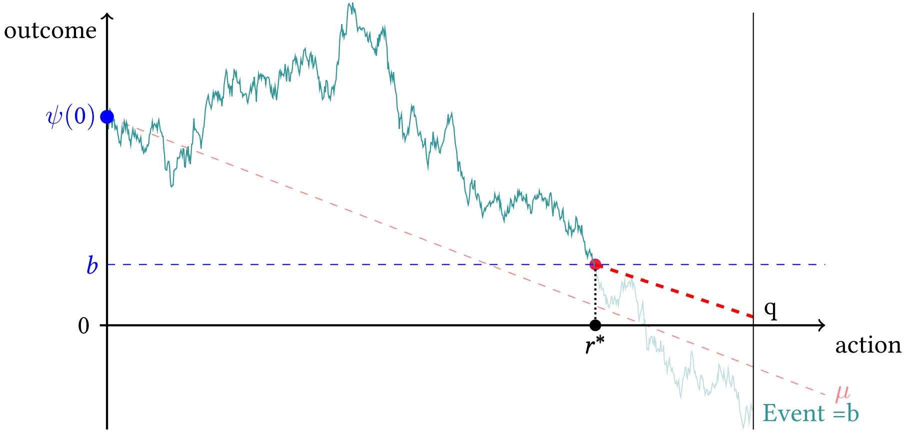

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Details for Event=b

- r^* is the first-minimum.
- No informational spillover to the right beyond $\psi(r^*) = b$ – Beliefs are neutral.

Details for Beliefs

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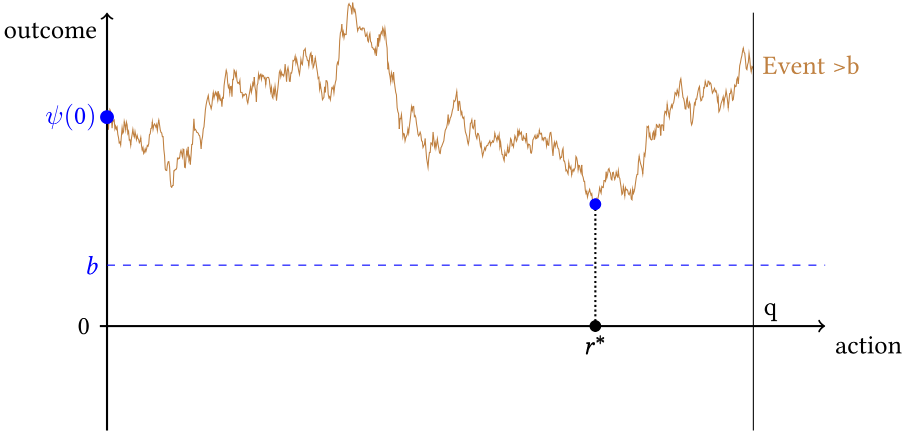

2. Event $> b$: Set of paths where $\psi(r^*) > b$.

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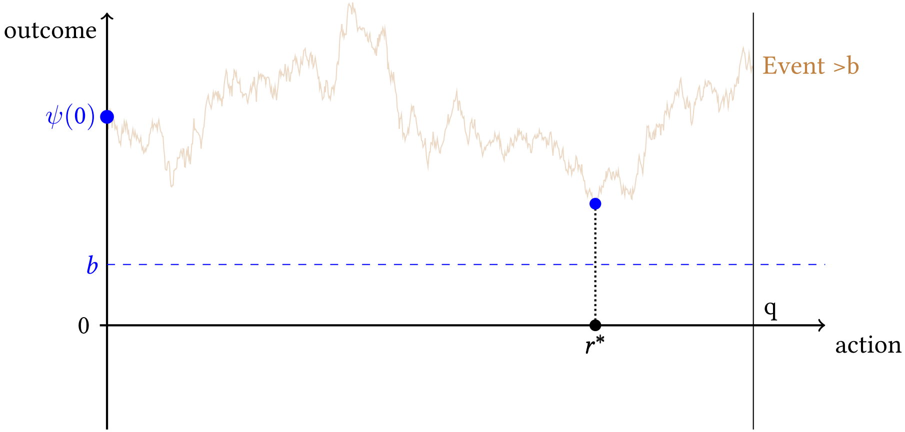

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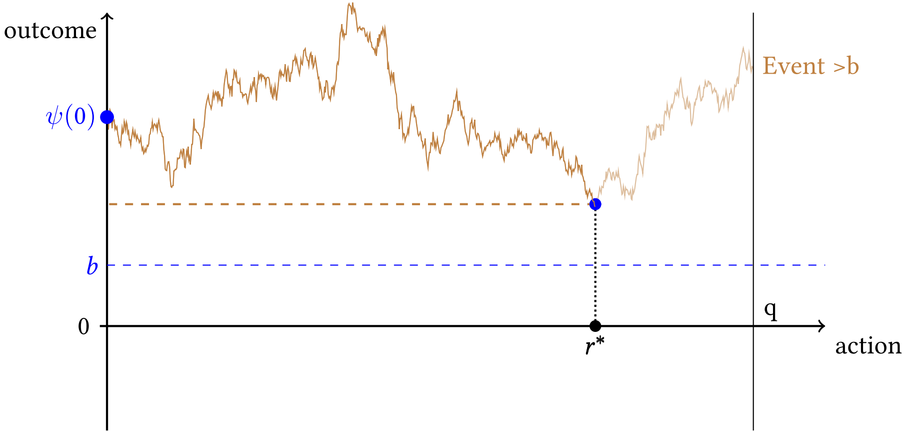
Details for Event>b

The Receiver's Inference Problem



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- r^* is the first-minimum.

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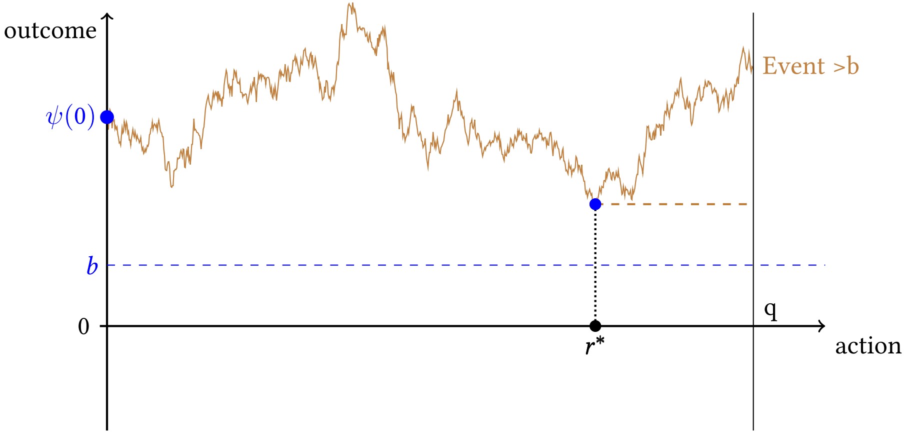

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- ▶ r^* is the first-minimum. And r^* is also the last-minimum.

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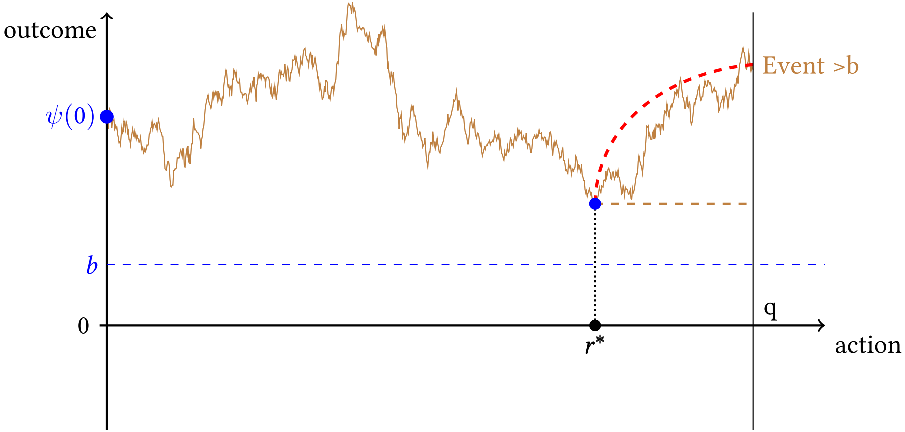

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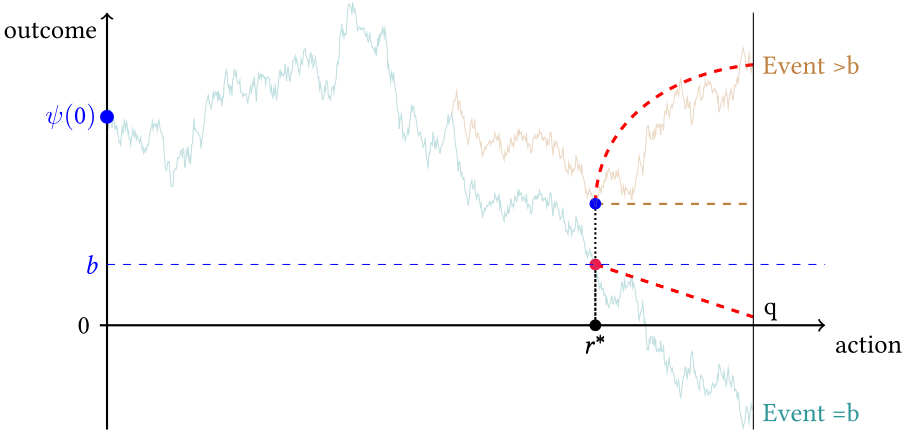
Details for Event>b

- r^* is the first-minimum. And r^* is also the last-minimum.
- Beliefs to the right are not neutral — Formally they follow a Brownian Meander process.

Details for Beliefs

The Receiver's Inference Problem



- Recommendation reveals precisely the sender's optimal action but imprecisely its outcome.
- Receiver forms posterior over these events using the Bayes' rule.

Details of Bayes Updating

- A new identity: The joint distribution of the hitting time and the location of the minimum.

Efficient Cheap Talk

Theorem 1. The first-point equilibrium exists if and only if $q \leq q_b^{\max}$:

(i) The misalignment is small compared to the risk complexity:

$$\text{If } 0 < b < \frac{\sigma^2}{2|\mu|} = \alpha \text{ then } q_b^{\max} = \infty.$$

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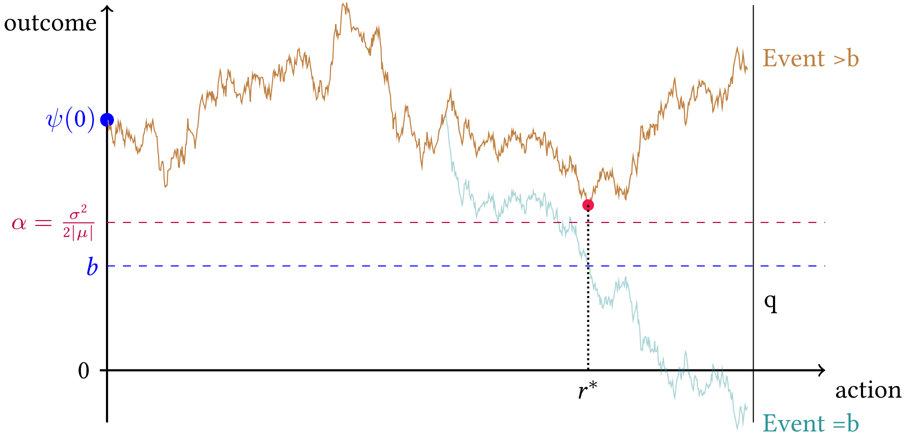

- Event $> b \implies$ Receiver and sender have aligned action preferences.

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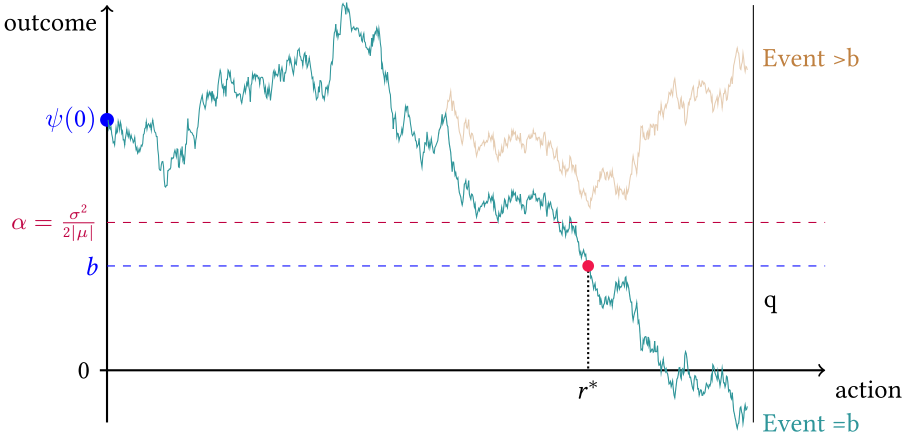

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- Event $= b \implies$ Receiver and sender have misaligned action preferences.

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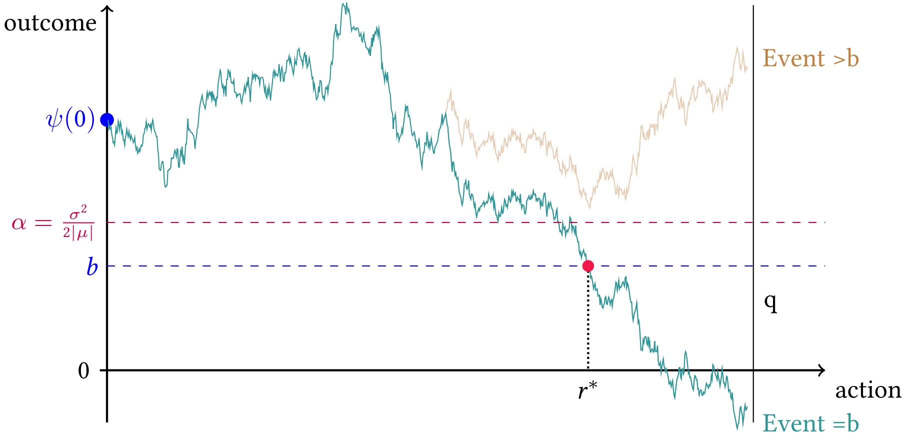

- Event $> b \implies$ Receiver and sender have aligned action preferences.
- Event $= b \implies$ Receiver and sender have misaligned action preferences.
- Logic of the 'no expert' result applies for deviations to right of the recommendation.

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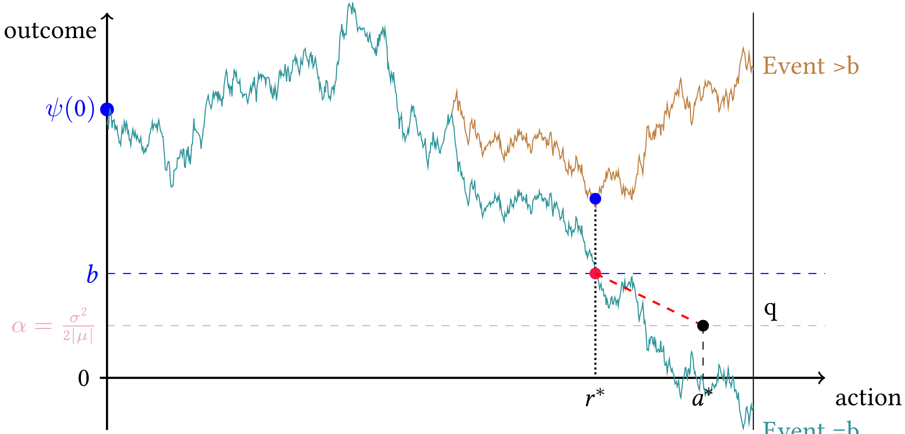
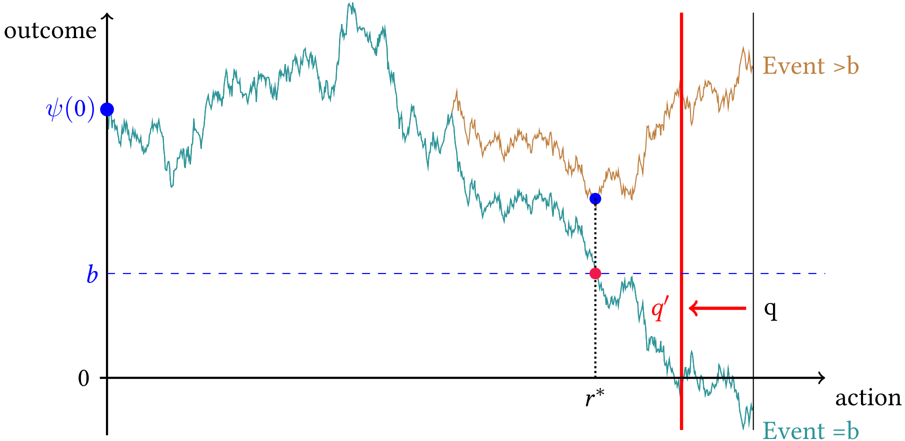
The Receiver's Optimal Response: $b > \alpha$



- For $b > \alpha$, the receiver faces a trade-off:
  - In Event >b the receiver's best response is $a = r^*$.
  - In Event =b the receiver's best response is a^* such that $\mathbb{E}[\psi(a^*)] = \alpha$.
- Efficient cheap talk requires the receiver to choose exactly r^* and nothing in between.

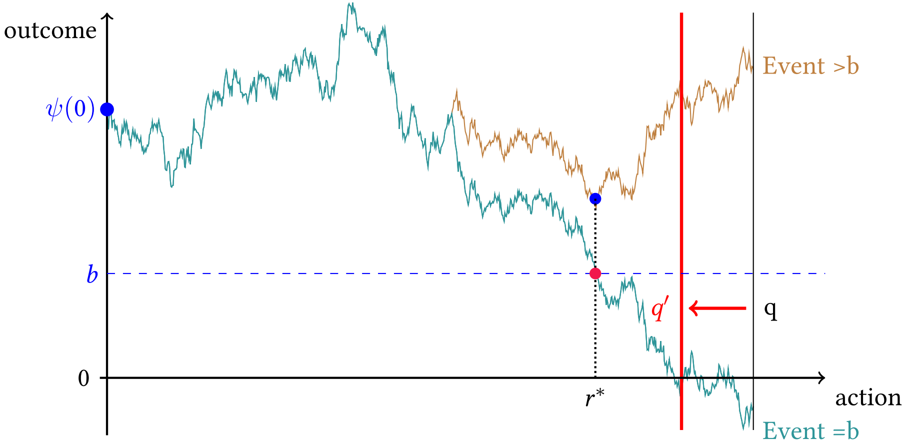
Equilibrium Dominance



Before showing existence, we first explain how action space influences the receivers inference problem.

Lemma 2: If the first-point equilibrium exists for $\mathcal{A} = [0, q]$, then it exists for $\mathcal{A}' = [0, q']$ whenever $q' < q$.

Equilibrium Dominance



- Same first-minimum and weaker last-minimum requirement as $q \to q'$.
  - No paths are eliminated as $q \to q'$.

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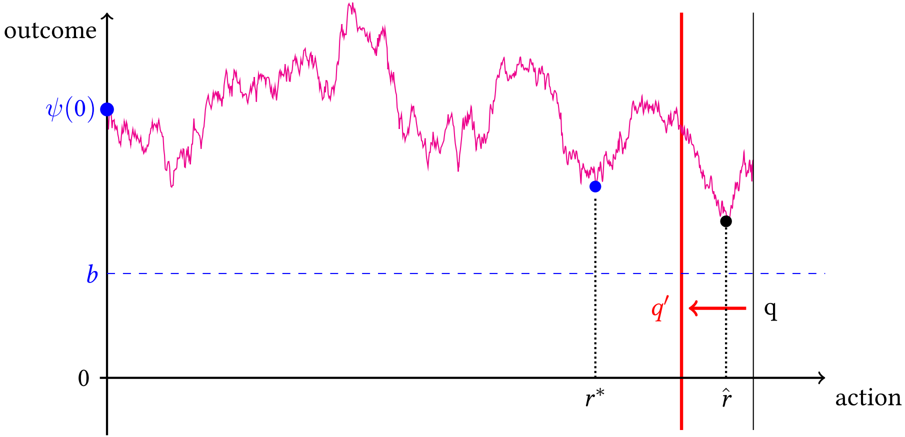

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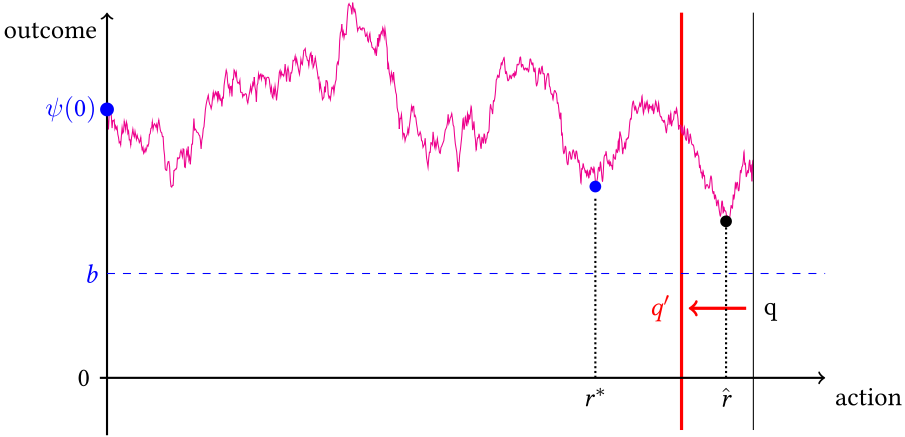

- Same first-minimum and weaker last-minimum requirement as $q \to q'$.
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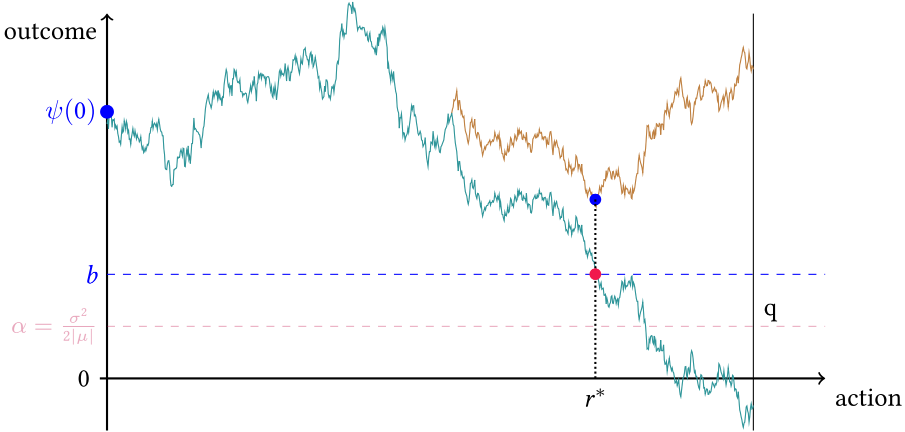
$\Rightarrow$ Probability of Event $> b$ is decreasing in q.

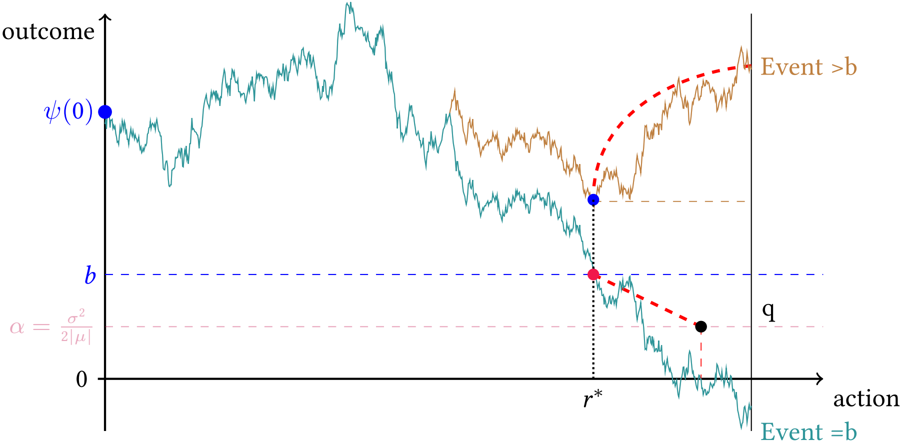
Details of the Argument

Equilibrium Existence



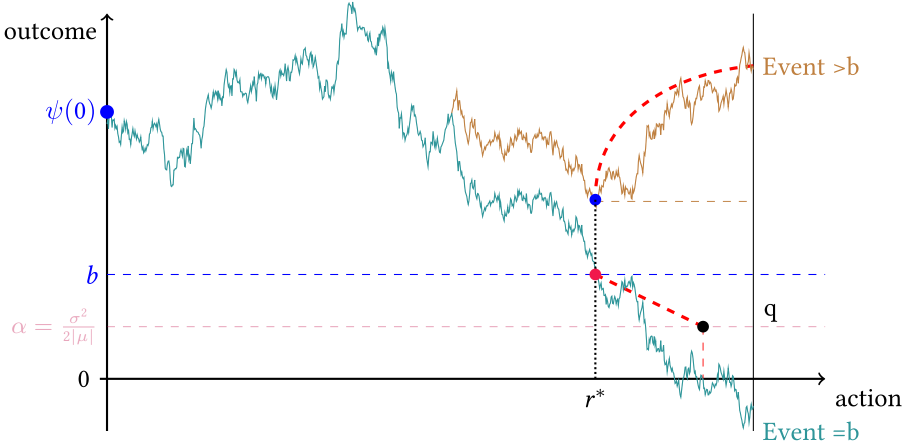
- **Lemma 3:** The first-point equilibrium exists for some $\mathcal{A} = [0, q]$ with $q > 0$.

Equilibrium Existence



- If q is not too large, the probability of Event $> b$ is greater than Event $= b$. Outline of the Proof
- Moreover, if q is not too large the Brownian Meander dominates in expectation. Moments of Brownian Meander

Equilibrium Existence



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Outline of the Proof

- Moreover, if q is not too large the Brownian Meander dominates in expectation.

Moments of Brownian Meander

- Equilibrium does not rely on risk aversion, although it makes achieving it easier.

Efficient Cheap Talk

Theorem 1. The first-point equilibrium exists if and only if one of the following holds:

(i) Risk complexity is high and the expert's recommendation is very hard to invert.

$$b \leq \frac{\sigma^2}{2|\mu|}$$

(ii) The action space is not too large and there is sufficient action alignment with the expert.

$$\mathcal{A} = [0, q] \text{ with } q \leq q_b^{\max}.$$

Comparative Statics I

How does the size of largest action space $q_b^{\max}$ change with the primitives of the environment?

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C1: $q_b^{\max}$ is decreasing in b for $b > \alpha$.

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- More likely to be 'aligned' (Event $> b$), and incremental gains are less attractive.

Increased σ has conflicting effects.

Detailed effect of σ

- Our simulations suggests that $q_b^{\max}$ is generally increasing in $\sigma > 0$.

Comparative Statics II

We also look for comparative statics for welfare within the first-point equilibrium (valid for $q < q_b^{\max}$).

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C4: Receiver utility strictly decreases and sender utility strictly increases in b.

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C5: Expected equilibrium outcome approaches to b as $\sigma \to \infty$.

- Very complex issues $\Rightarrow$ More likely to cross b & Receiver doesn't override $\Rightarrow$ Both better off.

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  $\Rightarrow$ Positive probability that the $\psi(\cdot)$ attains a minimum weakly above b at some $a \in \mathcal{S}$.

  $\Rightarrow$ Recommending the minimum instead improves the payoff for both players.

- Full support action distribution $\Rightarrow$ Sender recommends own most preferred action.

1. The Model: Introducing Complex Environments.
2. Results
   - Decision making without the Expert.
   - Main Result: Decision making with the Expert.
3. Extensions: Brownian Motion and Beyond.

Delegating v. Communicating?

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- Receiver can commit to taking actions from $[0, q_b^{\max}]$ to facilitate efficient communication.
- But it is better for him to delegate full decision making power.
  - Restriction to $[0, q_b^{\max}]$ creates action-alignment by making both players worse off.

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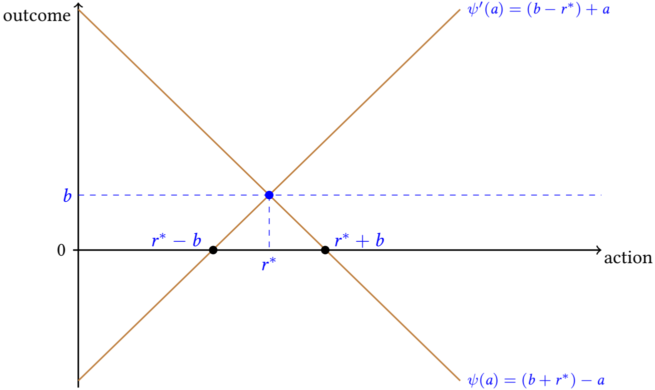
Expert reveals her own optimal action $\not\Rightarrow$ Decision maker learns his optimal action.

Minimally Complex Extension of Crawford-Sobel



- Outcome Mapping: $\psi(a) = \psi_0 + a$ or $\psi(a) = \psi_0 - a$ with $\psi_0 \in \mathbb{R}$.

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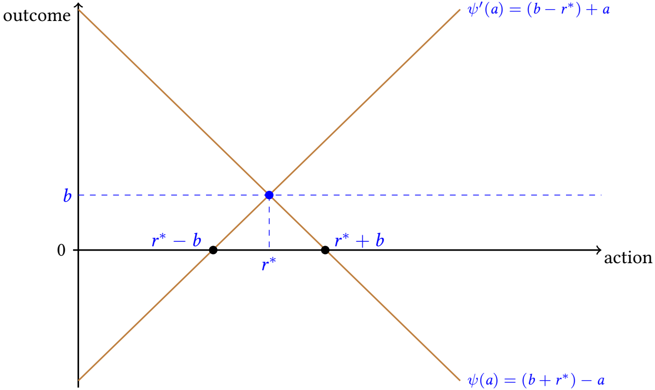

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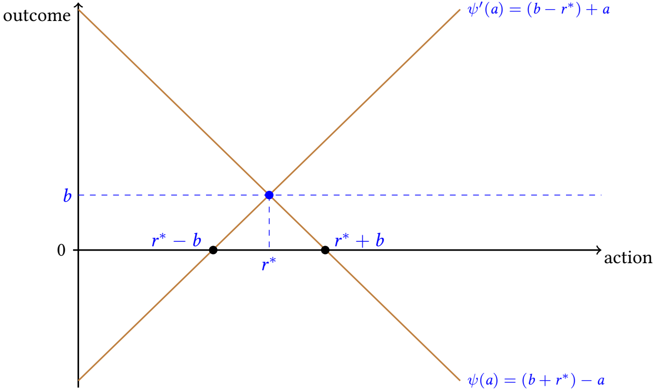

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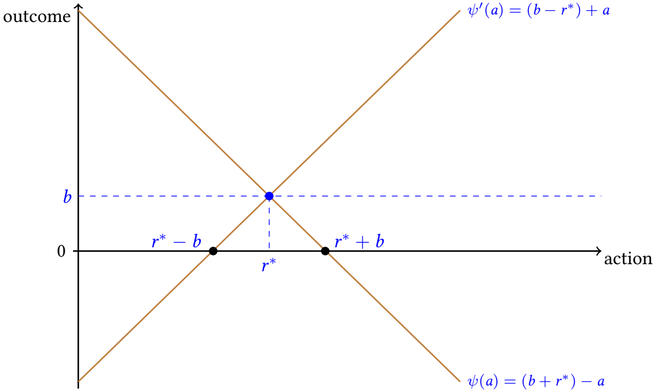

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- Efficient equilibrium exists if and only if two states are *equally* likely.
- Players always have different optimal actions. Receiver faces directional uncertainty.

Expert Advice in the Long-run

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- Decision maker learns the relationship between actions and outcomes over time.
- Expert can't use her information efficiently – communicates inefficiently to keep the receiver uncertain.
- Decision maker's ability to learn can make communication so inefficient that she becomes worse off compared to single-period efficient communication.

Literature Review

- Cheap Talk (Crawford and Sobel, 1982).
  - Invertible. Equilibrium: Expert sacrifices power to make recommendations non-invertible.
- Bayesian Persuasion (Kamenica and Gentzkow, 2011).
  - Commitment makes recommendation non-invertible. We get sender-optimal without commitment.
- Unknown bias (Morgan and Stocken, 2003).
  - Non-invertible, but low residual uncertainty $\Rightarrow$ Equilibria are generally inefficient.
- Discrete and Independent Actions (Aghion and Tirole, 1997)
  - No informational spillover.
- Brownian Motion (Callander, 2008; Callander, Lambert, Matouschek, 2021; Dall'Ara, 2023).
  - We study non-invertibility broadly, and how sender can shape information spillover.

Conclusion

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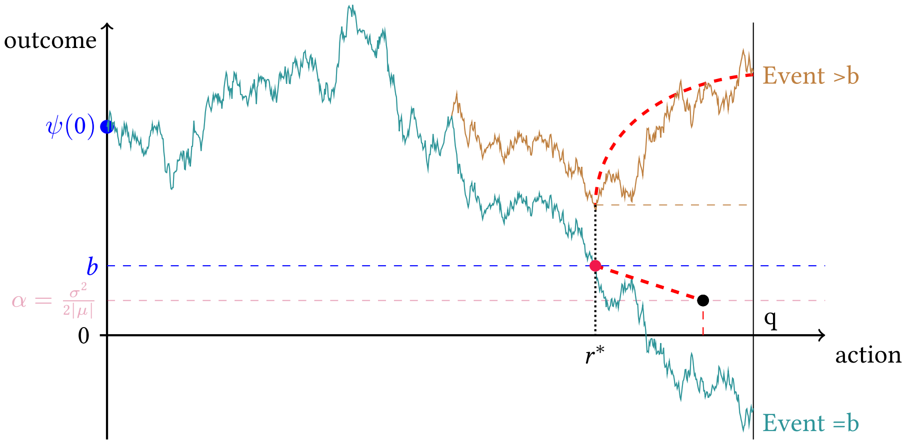
- In practice experts are "overtowering." In models they have no power and communication is inefficient.
- We develop a novel and comprehensive framework that captures expert-power in practice.
- When the decision maker faces large "information inequality" the canonical results are reversed:
  1. Experts have full power – they can implement her optimal action in equilibrium.
  2. Communication is efficient.
- Expert power comes from how much information remains private after the recommendation.


outcome
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b
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0
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action
Event >b
Event =b

Thank You!

Extra Slides

States and Beliefs

Back to the Model

- $\psi : A \to \mathbb{R}$ and Ψ is the set of all ψ.
- It can be also thought as if $\psi(\cdot)$ is a known function of a random variable θ (with underlying probability triple $(\Omega, \mathcal{F}, \omega)$) privately observed by the sender.
- State is θ and state space is $\theta \in \Theta$.
- Receiver prior belief: $\omega(\cdot)$ over Θ
  - e.g. $\Theta = [0, 1]$ and ω is the uniform distribution.
  - e.g. $\theta = C[0, q]$ and ω is the Wiener measure.
- We refer to the induced beliefs about $\psi(\cdot)$ instead of ω.

We call $\omega(\cdot \mid \cdot)$, $a(\cdot)$, $m(\cdot)$ a Perfect Bayesian Equilibrium if

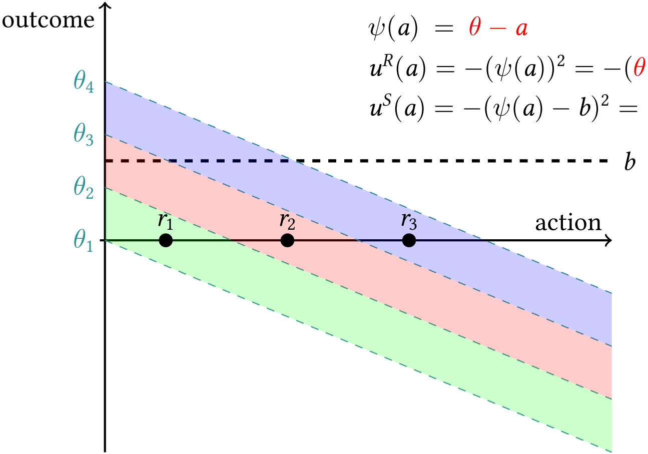
1. $\omega(\psi \mid r \in m(\psi))$ is obtained via Bayes' rule whenever possible,
2. $a(r) \in \arg\max_{a' \in \mathcal{A}} \mathbb{E}[u_R(a', \psi) \mid \omega(\psi \mid r \in m(\psi))]$ for every $r \in \mathcal{M}$,
3. $m(\psi) \in \arg\max_{r' \in \mathcal{M}} u_S(a(r'), \psi)$ for every $\psi \in \Psi$.

Simple Environments

Back to Simple Environments

- Players: Sender and Receiver.
- Actions: $\mathcal{A} = \mathbb{R}_+$.
- Outcomes: $\psi(a) = \theta - a$ common knowledge
- Sender's private information: realized θ.
- Receiver's prior: $\theta \sim \mathcal{I} \subseteq \mathbb{R}_+$.
- Payoffs: $u^S(a) = -(\psi(a) - b)^2 = -(\theta - a - b)^2$, $u^R(a) = -(\psi(a))^2 = -(\theta - a)^2$.

Simple Environments: Equilibrium



$$\psi(a) = \theta - a$$
$$u^R(a) = -(\psi(a))^2 = -(\theta - a)^2$$
$$u^S(a) = -(\psi(a) - b)^2 = -(\theta - a - b)^2$$

- All equilibria are partitional: $m^*(\theta) = r_i$ if and only if $\theta_i \in [\theta_{i-1}, \theta_i]$.
- Sender incentive compatibility limits the number of partitions.
- If partitions are too small, types at the boundary are too close to each other.

Back to Simple Environments

Complex Environments

Back to Complex Environments

- Players: Sender and Receiver.
- Actions: $\mathcal{A} = \mathbb{R}_+$.
- Outcomes:$\psi(a) = \psi_0 + \mu a + \sigma W(a)$.
  - The parameters ψ_0, μ and σ common knowledge.
- Formally state is $W(a)$ and state space is $\mathcal{C}[0, q]$.
- Sender's private information: The realized path $\psi(a)$.
- Receiver prior belief: $\omega(\cdot)$ over $\mathcal{C}[0, q]$ given by the Wiener measure.
  - We generally refer to the induced beliefs about $\psi(\cdot)$ instead of $W(\cdot)$.

Proof of Lemma 1

By the mean-variance representation of quadratic utility, the receiver's expected utility is:

$$\mathbb{E}[u_R(a)] = -\left[\psi(0) + \mu a\right]^2 - \sigma^2 a.$$

The first and second order conditions for optimality are:

$$\frac{d\mathbb{E}[u_R(a)]}{da} = -2\mu\left[\psi(0) + \mu a\right] - \sigma^2,$$
$$\frac{d^2\mathbb{E}[u_R(a)]}{da^2} = -2\mu^2 \leq 0.$$

The result follows from the first order condition. ∎

- We get a similar result for other weakly concave utility.
- But α is no longer a constant threshold.

Back to No Expert

Probabilities of Event $= b$

- We can define Event$= b$ using the hitting "action" (time).

Back to Event=b

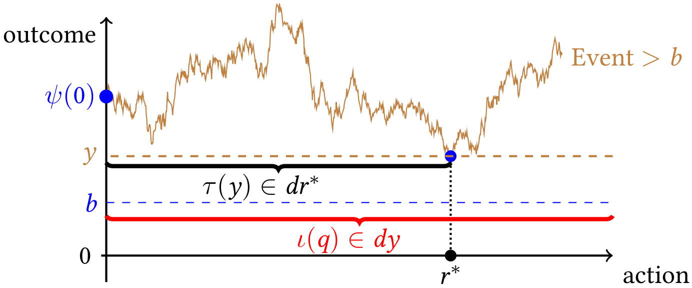
- First hitting action: $\tau(x) := \inf\{a \in [0, q] \mid \psi(a) = x\}$.
- Probability of the path first-hitting $b < \psi_0$:

$$\mathbb{P}(\text{Event} = b \text{ at } a) = \mathbb{P}(\tau(b) \in da) = \frac{\psi_0 - b}{\sigma a \sqrt{a}} \phi\left(\frac{\psi_0 - b + \mu a}{\sigma \sqrt{a}}\right) da \quad \forall x \in \mathbb{R}_+.$$

Probabilities of Event $> b$

Back to Event>b

- First hitting action: $\tau(x) := \inf\{a \in [0, q] \mid \psi(a) = x\}$.
- Minimum of the path $\iota(w, x)$: $\iota(w, x) = \inf\{\psi(a) \mid a \in [w, x]\}$.
- $\mathbb{P}(\text{Event} > \text{b at } m^*(\psi) = r^*) = \int_b^{\psi_0} \mathbb{P}(\tau(y) \in dr^*, \iota(q) \in dy)dy$.

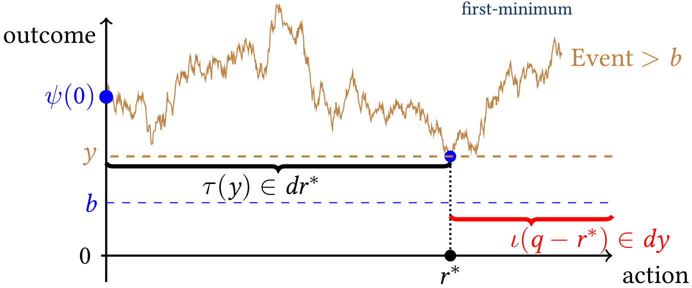


Probabilities of Event $> b$

Back to Event>b

- First hitting action: $\tau(x) := \inf\{a \in [0, q] \mid \psi(a) = x\}$.
- Minimum of the path $\iota(w, x)$: $\iota(w, x) = \inf\{\psi(a) \mid a \in [w, x]\}$.
- $\mathbb{P}(\text{Event} > \text{b at } m^*(\psi) = r^*) = \int_b^{\psi_0} \mathbb{P}(\tau(y) \in dr^*, \iota(q) \in dy) dy$.
- Using the Strong Markov Property of $W(a)$:

$$\mathbb{P}(\text{Event} > \text{b at } m^*(\psi) = r^*) = \int_b^{\psi(0)} \underbrace{\mathbb{P}\{\tau(y) \in dr^*\}}_{\text{first-minimum}} \cdot \underbrace{\mathbb{P}\{\iota(r^*, q) \in dy\}}_{\text{last-minimum}} dy$$



Bayes Updating

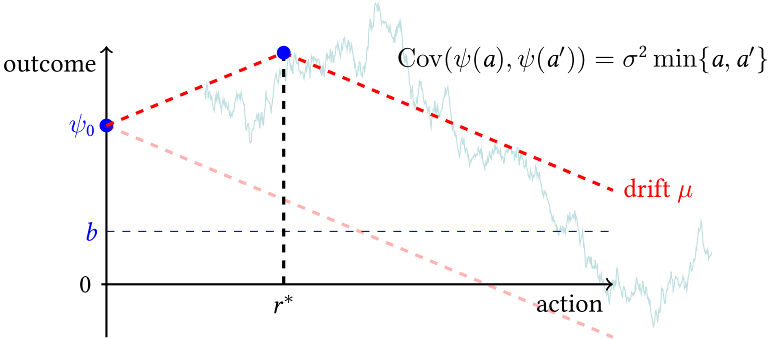
Back to Receiver's Inference

- We are interested in $\mathbb{P}(\text{Event} = \text{b} \mid m^*(\psi) = r^*)$.
- Conditioning event $m^*(\psi) \in dr^*$ is the (disjoint) union of two events:
  1. Event =b at $m^*(\psi)$,
  2. Event>b at $m^*(\psi)$.
- Regular conditional probability can be obtained as follows:

$$\begin{aligned}
\mathbb{P}(\text{Event} = \text{b} \mid m^*(\psi) = r^*) &= \frac{\mathbb{P}(\text{Event =b at } m^*(\psi) \in dr^*)}{\mathbb{P}(m^*(\psi) = r^*)} \\
&= \frac{\mathbb{P}(\text{Event =b at } m^*(\psi) \in dr^*)}{\mathbb{P}(\text{Event =b at } m^*(\psi) \in dr^*) + \mathbb{P}(\text{Event>b at } m^*(\psi) \in dr^*)} \\
&= \frac{\mathbb{P}(\tau(b) \in dr^*)}{\mathbb{P}(\tau(b) \in dr^*) + \int_b^{\psi_0} \mathbb{P}(\tau(y) \in dr^*)\mathbb{P}(\iota(r^*, q) \in dy)dy}
\end{aligned}$$

- Densities are well defined everywhere $r^* \in (0, q]$.

Brownian Motion: Conditional Beliefs



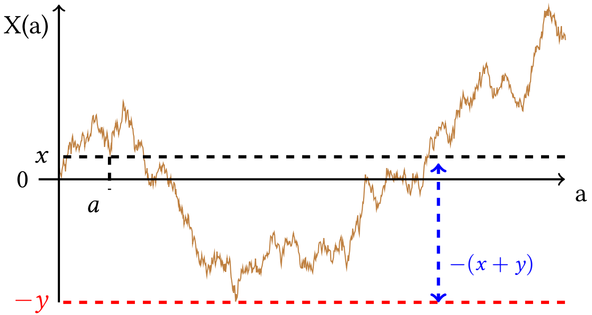
- The beliefs conditional on $\psi(r^*) = y$ are:

Back to Complex Environments | Back to Inference

$$\mathbb{E}[\psi(a)|\psi(r^*) = y] = \begin{cases} \psi_0 + \frac{a}{r^*}(y - \psi_0) & \text{if } a \leq r^* \\ y + \mu a & \text{if } a \geq r^* \end{cases}$$

$$\mathrm{Var}[\psi(a)|\psi(r^*) = y] = \begin{cases} \sigma^2 \frac{a(r^* - a)}{r^*} & \text{if } a \leq r^* \\ \sigma^2(a - r^*) & \text{if } a \geq r^* \end{cases}$$

Brownian Meander I



- Rescale such that $X(a) = \psi(a) - \psi_0 = \mu a + \sigma W(a)$.

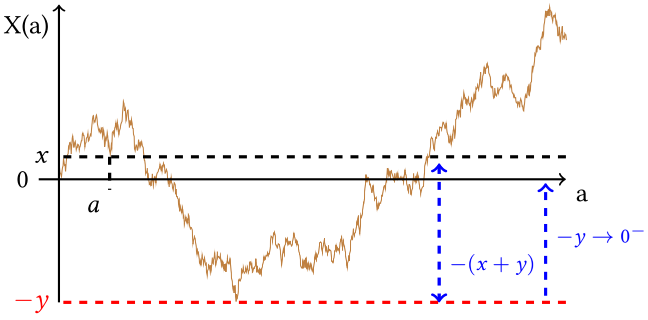
Back to Inference
Back to Existence

$$\mathbb{P}(X(a) \in dx \mid \iota(q) \geq -y) = \frac{\mathbb{P}(X(a) \in dx, \iota(q) \geq -y)}{\mathbb{P}(\iota(q) \geq -y)}$$

$$= \frac{\mathbb{P}(X(a) \in dx, \iota(a) \geq -y, \iota(q-a) \geq -(x+y))}{\mathbb{P}(\iota(q) \geq -y)}$$

$$\mathbb{P}(X(a) \in dx \mid \iota(q) \geq -y) = \frac{\mathbb{P}(X(a) \in dx, \iota(a) \geq -y)\mathbb{P}(\iota(q-a) \geq -(x+y))}{\mathbb{P}(\iota(q) \geq -y)}.$$

Brownian Meander II



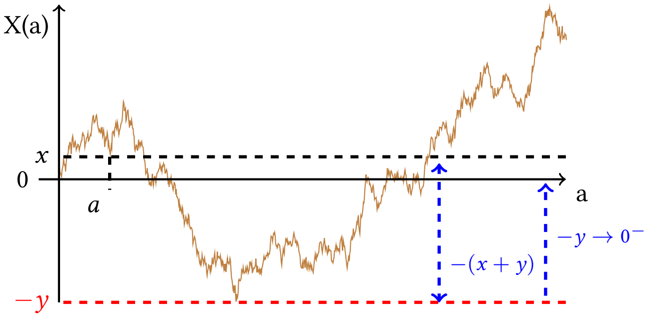
- Looking at $\lim_{-y \to 0^-} \mathbb{P}(X(a) \in dx \mid \iota(q) \geq -y)$:

Back to Inference Back to Existence

$$\mathbb{P}(M(a,q) \in dx) = \frac{\sqrt{q}x}{\sigma a \sqrt{a}} \frac{\exp\left(\frac{\mu^2 q}{2\sigma^2}\right) \phi\left(\frac{\mu a - x}{\sigma\sqrt{a}}\right) \left(\Phi\left(\frac{x+\mu(q-a)}{\sigma\sqrt{q-a}}\right) - \exp\left(-\frac{2\mu x}{\sigma^2}\right) \Phi\left(\frac{-x+\mu(q-a)}{\sigma\sqrt{q-a}}\right)\right)}{\left(\mu\sqrt{q}\exp\left(\frac{\mu^2 q}{2\sigma^2}\right) \Phi\left(\frac{\mu\sqrt{q}}{\sigma}\right) + \frac{\sigma}{\sqrt{2\pi}}\right)} dx.$$

- Details of the weak convergence follows from standard arguments.
- See Durrett et al. (1977) and Iafrate and Orsingher (2020) for the details.

Brownian Meander II



- Looking at $\lim_{-y\to 0^-} \mathbb{P}(X(a) \in dx \mid \iota(q) \geq -y)$: Back to Inference Back to Existence

$$\mathbb{P}(M(a,q) \in dx) = \frac{\sqrt{q}x}{\sigma a\sqrt{a}} \frac{\exp\left(\frac{\mu^2 q}{2\sigma^2}\right)\phi\left(\frac{\mu a - x}{\sigma\sqrt{a}}\right)\left(\Phi\left(\frac{x+\mu(q-a)}{\sigma\sqrt{q-a}}\right) - \exp\left(-\frac{2\mu x}{\sigma^2}\right)\Phi\left(\frac{-x+\mu(q-a)}{\sigma\sqrt{q-a}}\right)\right)}{\left(\mu\sqrt{q}\exp\left(\frac{\mu^2 q}{2\sigma^2}\right)\Phi\left(\frac{\mu\sqrt{q}}{\sigma}\right) + \frac{\sigma}{\sqrt{2\pi}}\right)} dx.$$

- It coincides with equation (1.4) in Iafrate and Orsingher (2020) when $\sigma = 1$.
- It coincides with Rayleigh distribution whenever $\mu = 0$, $\sigma = 1$ and $a = q$.

Moments of Brownian Meander

Back to Existence

- We characterize the distribution of $M(a,q)$ given its terminal value.
- Special case of $\mu = 0$ and $\sigma = 1$ is analyzed in Devroye (2010) and Riedel (2021).

- This is obtained by the limit: $\lim_{-y\to 0^-} \mathbb{P}(X(a) \in dx \mid X(q) = c, \iota(q) \geq -y)$:

$$\mathbb{P}(M(a,q) \in dx \mid M(q,q) = c) = \frac{xq\sqrt{q}}{ca\sqrt{a}\sqrt{q-a}\sigma}\left[\phi\left(\frac{x - \frac{ca}{q}}{\sqrt{\frac{a}{q}}\sqrt{q-a}\sigma}\right) - \phi\left(\frac{x + \frac{ca}{q}}{\sqrt{\frac{a}{q}}\sqrt{q-a}\sigma}\right)\right] dx$$

$$\mathbb{E}[M(a,q)|M(q,q) = c] = \frac{\sigma^2(q-a) + \frac{c^2 a}{q}}{c}\,\mathrm{erf}\left(\frac{c\sqrt{a}}{\sigma\sqrt{2q(q-a)}}\right) + \exp\left(\frac{-c^2 a}{2q(q-a)\sigma^2}\right)\sqrt{\frac{2a(q-a)}{q\pi}}\sigma$$

$$\mathbb{E}[M^2(a,q) \mid M(q,q) = c] = \frac{3(q-a)a}{q}\sigma^2 + \frac{c^2 a^2}{q^2}$$

- It follows that that $\lim_{a\to 0^+} \frac{\partial}{\partial a}\mathbb{E}[M(a,q)|M(q,q) = c] = \infty$.

Equilibrium Dominance

Back to Lemma 2

- First hitting action: $\tau(x) := \inf\{a \in [0, q] \mid \psi(a) = x\}$.
- Minimum of the path $\iota(w, x)$: $\iota(w, x) = \inf\{\psi(a) \mid a \in [w, x]\}$.
- We have the probabilities given by:

$$\mathbb{P}(\text{Event} = b \text{ at } r^*) = \mathbb{P}(\tau(b) \in dr^*)) = \frac{\psi_0 - b}{\sigma r^* \sqrt{r^*}} \phi\left(\frac{\psi_0 - b + \mu r^*}{\sigma \sqrt{r^*}}\right) dr^* \quad \forall x \in \mathbb{R}_+$$

$$\mathbb{P}(\text{Event >b at } r^*) = \int_b^{\psi(0)} \underbrace{\mathbb{P}\{\tau(y) \in dr^*\}}_{\text{first-minimum}} \cdot \underbrace{\mathbb{P}\{\iota(r^*, q) \in dy\}}_{\text{last-minimum}} dy.\mathbb{P}\{\iota(r^*, q) \in dy\}$$

- As q gets smaller, $\tau(b) \in dr^*$ is constant and $\mathbb{P}\{\iota(r^*, q) \in dy\}$ is increasing.
- Thus, $\mathbb{P}(\text{Event =b} \mid m^*(\psi)$ decreasing:

$$\mathbb{P}(\text{Event =b} \mid m^*(\psi) = r^*) = \frac{\mathbb{P}(\text{Event =b at } m^*(\psi) \in dr^*)}{\mathbb{P}(\text{Event =b at } m^*(\psi) \in dr^*) + \mathbb{P}(\text{Event>b at } m^*(\psi) \in dr^*)}.$$

Equilibrium Existence

Change in **expected outcome** for a deviation to $r^* + a'$ is given by:

Back to Existence

$$\Delta(a', r^*, q) = \mathbb{P}(\text{Event} = \text{b} \mid m^*(\psi) = r^*)(\mu a') + \mathbb{P}(\text{Event} > \text{b} \mid m^*(\psi) = r^*)\mathbb{E}[M(a', q - r^*)]$$

1. We showed that $\mathbb{P}(\text{Event} = \text{b} \mid m^*(\psi) = r^*)$ decreasing.
2. Moreover, $\mathbb{P}(\text{Event} = \text{b} \mid m^*(\psi) = r^*) \to 1$ for every as $r^* \to 0$.
3. We show that $\lim_{a \to 0^+} \frac{\partial}{\partial a}\mathbb{E}[M(a, q)|M(q, q) = c] = \infty$ for every q^*.

- If $q \to 0$, then $\max\{a, r^*\} \to 0$. So $\lim_{q \to 0} \Delta(a', r^*, q) > 0$
- Thus, for some $\bar{q} > 0$ we have that $\Delta(a', r^*, \bar{q})$ for every $a', r^*, q < \bar{q}$.
- Note that $\bar{q} \neq q^b_{\max}$: $q^b_{\max}$ is the largest solution q is the counterpart for **expected payoff**.

Action Space v. Complexity

- We develop our analysis by varying the size of the action space instead of σ or α. Back to Size of the Action Space

- Expert derives power from the complexity of the environment but not in direct proportion to complexity.

- Increased σ has conflicting effects.

1. Changes what the receiver infers from the recommendation
   - Probability of Event$=b$ is non-monotone, and increasing on average.
   - Makes it harder to support the equilibrium.
2. Changes the shape of receiver uncertainty about other actions.
   - Expectations for deviations in Event $> b$ becomes more steep.
   - Riskiness of deviations increase in both events.
   - Makes it easier to sustain.

- Drift μ closer to 0 also decreases the probability of Event $> b$.
  - Equilibrium is always easier to support.

Extensions: Robustness within BM

Extensions within Brownian Motion

- **Weakly concave utility with an unique maximum.** Details for the Extension
  - The α threshold is not a constant – Everything else goes through.
- **Very large bias:** $b > \psi_0$. Demonstration of Large Bias
  - Interests are diametrically opposed, only equilibria are babbling.
- **Negative Bias:** $b < 0$. Demonstration of Negative Bias
  - In event $= b$, receiver knows there is an action to the left that gives his ideal.
- **Actions to the left of the status quo.** Demonstration of Actions to the Left
  - Recommendations to the left of status quo are easier to implement due to $\mu < 0$.

Beyond Quadratic Utilities

Back to Extensions

Sketch of the Idea

Say that the receiver's utility is separable in mean $\mu(a) = \mathbb{E}[\psi(a)]$ and variance $\sigma(a) = \text{Var}[\psi(a)]$:

$$\mathbb{E}[u_R(a)] = v(\mu(a)) - w(\sigma(a)).$$

The first and second order conditions for optimality are:

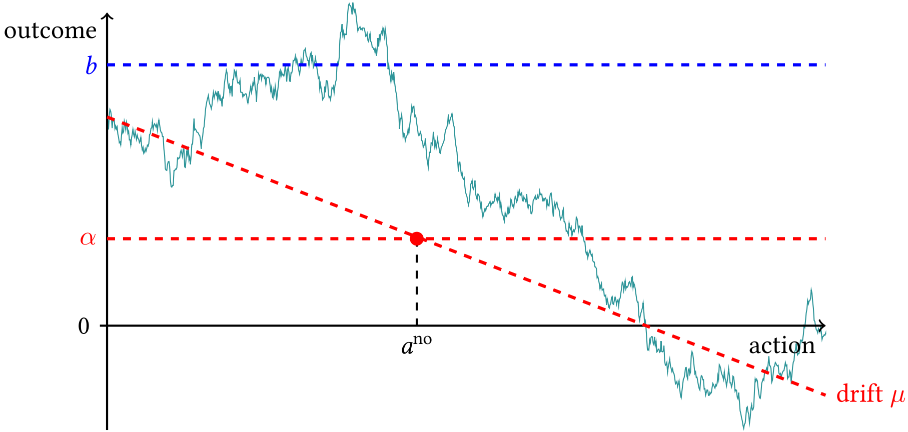
$$\frac{d\mathbb{E}[u_R(a)]}{da} = \mu'(a)v'(\mu(a)) - \sigma'(a)w(\sigma(a)) = 0$$

$$\frac{d^2\mathbb{E}[u_R(a)]}{da^2}\mu''(a)v'(\mu(a)) + \mu'(a)^2v''(\mu(a)) - \sigma''(a)w'(\sigma(a)) - \sigma'(x)^2w''(\sigma(x)) \leq 0$$

$$a = \mu^{-1}\left((v')^{-1}\left(\frac{\sigma'(a)v'(\sigma(a))}{\mu'(a)}\right)\right)$$

The result follows from the first order condition under suitable conditions on the curvature of $\mu(a)$ and $\sigma(a)$.
e.g. $\mu'(x) < 0$, $\mu''(x) \leq 0$ and $\sigma'(x) > 0$, $\sigma''(x) > 0$ and $w''(x), v''(x) \leq 0$ ■

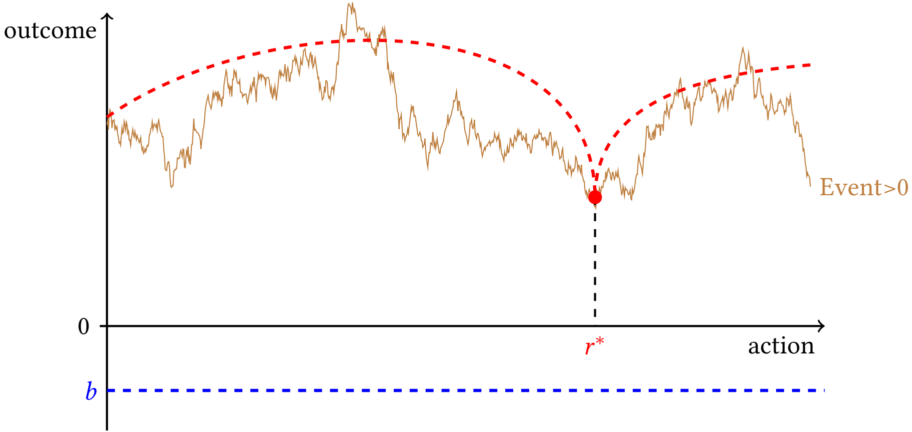
Very Large Bias



- Interests are fully misaligned.
  - If an outcome is better than the status quo for the sender is worse for the receiver.
- Only equilibria are babbling.

Back to Extensions

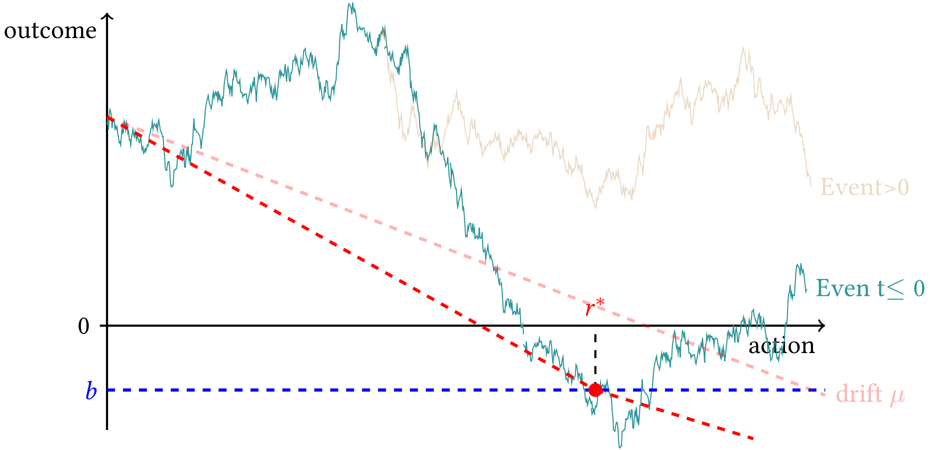
Negative Bias



- Event > 0 works the same way.

Back to Extensions

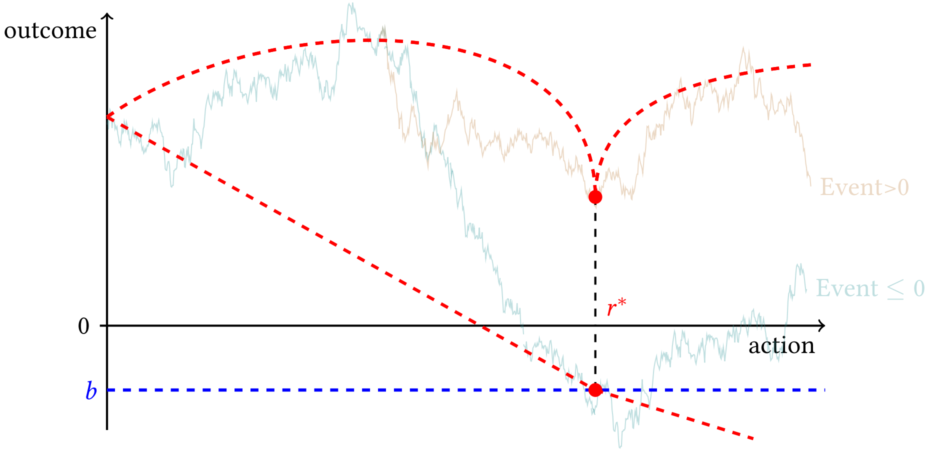
Negative Bias



- Event > 0 works the same way.
- In Event ≤ 0, now there is a profitable deviation is now to the left.

Back to Extensions

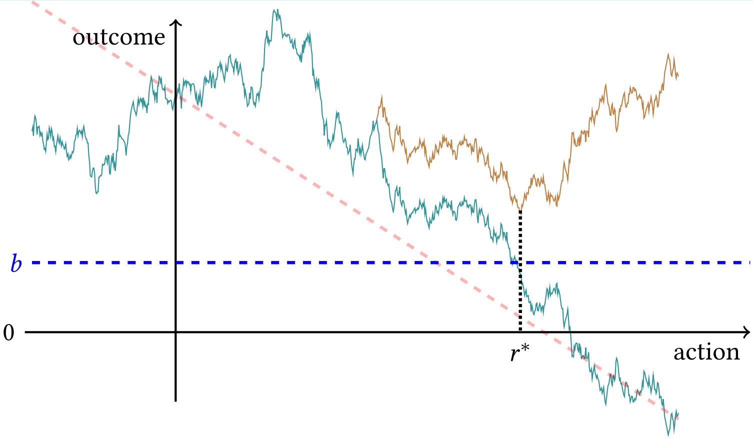
Negative Bias



- Event > 0 works the same way.
- In Event ≤ 0, now there is a profitable deviation is now to the left.
- A similar upper bound like q^b_{max} can be constructed.

Back to Extensions

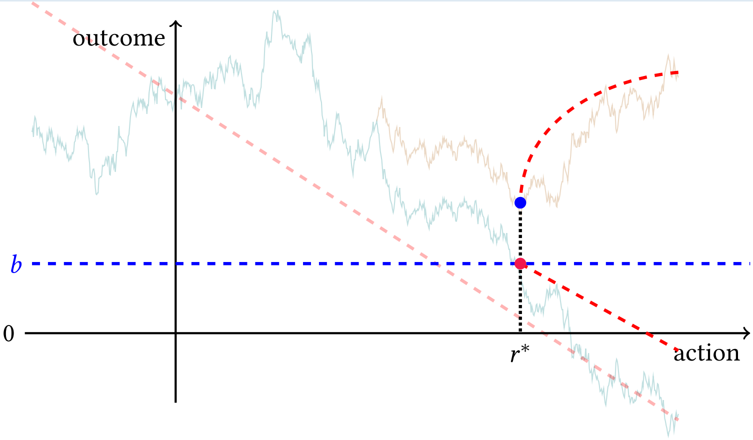
To the Left of Status quo



- If the recommendation is $r^* > 0$:

Back to Extensions

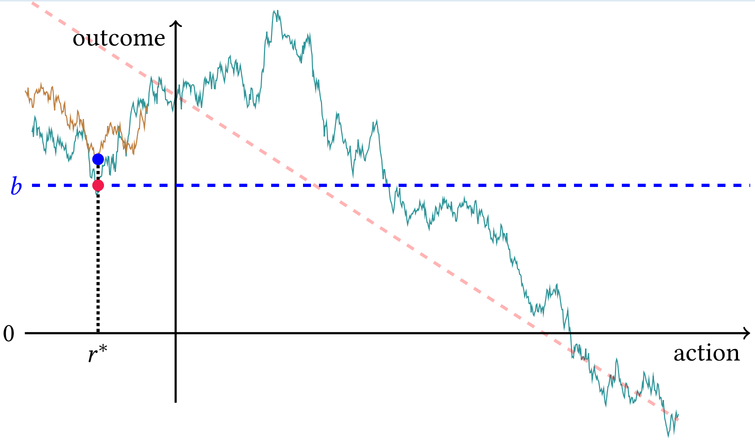
To the Left of Status quo



- If the recommendation is $r^* > 0$: It is the same problem and $q^b_{\max}$ works.

Back to Extensions

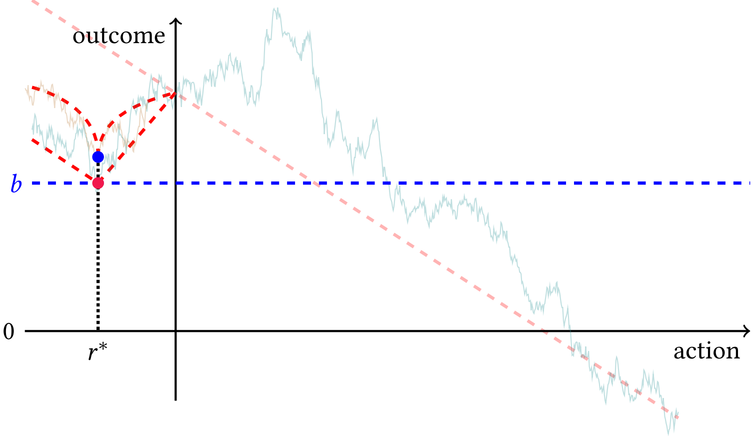
To the Left of Status quo



- If the recommendation is $r^* > 0$: It is the same problem and $q^b_{\max}$ works.
- If the recommendation is $r^* < 0$:

Back to Extensions

To the Left of Status quo



- If the recommendation is $r^* > 0$: It is the same problem and $q^b_{\max}$ works.
- If the recommendation is $r^* < 0$:
- Drift μ has the opposite effect and the Receiver IC is always satisfied when r^*.

Back to Extensions

Extensions: Examples Beyond BM

Conditions for Expert Power I

Suppose that the sender uses $m : \psi \to \mathcal{A}$ that *precisely reveals* his optimal action.

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$$|m^{-1}(r)| > 1 \quad \forall r \in \mathcal{A}$$

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- When does that lead the receiver to accept the sender's optimal action?

3. **Recommendation Acceptance:** Receiver's incentive compatibility is satisfied.

$$r \in \arg\max_{a \in \mathcal{A}} \mathbb{E}[u^R(a, \psi) \mid \psi \in m^{-1}(r)] \quad \forall r \in m^{-1}(\Psi)$$

Back to Extensions

Conditions for Expert Power II

1. **Partial Invertibility:** Multiple states are consistent with recommendation.
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- **First-point strategy** in Brownian Motion environment satisfies (1) and (2).
- **First-point strategy** also satisfies (3) if action space is narrow $\left(q < q^{b}_{\max}\right)$.

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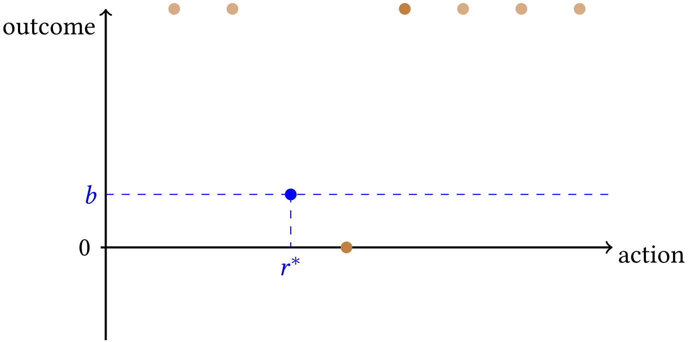
- **Efficient strategies** in canonical cheap talk fail (1).
- **Partition strategies** in canonical cheap talk satisfy (1) and (2).
- **Partition strategies** also satisfy (3) if partitions are large enough.

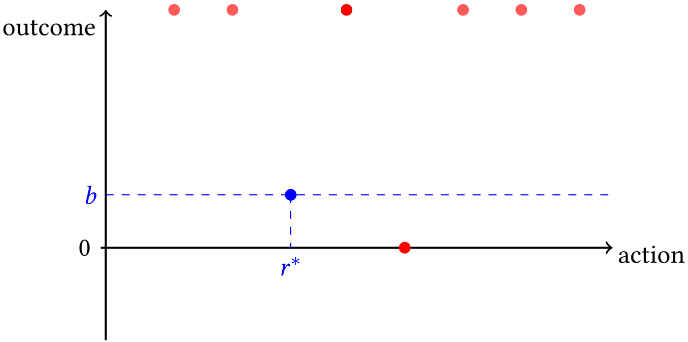
Back to Extensions

Misalignment Without Directional Uncertainty



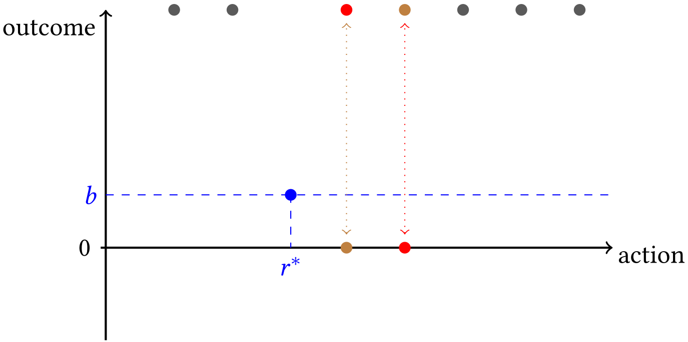
- For each $a \in \mathcal{A} = \mathbb{Z}$, there are two states ψ and ψ':
  - $\psi(a) = b$, $\psi(a+1) = 0$ and $\psi(a') = 100b \;\; \forall a' \in \mathcal{A} \setminus \{a, a+1\}$.

Misalignment Without Directional Uncertainty



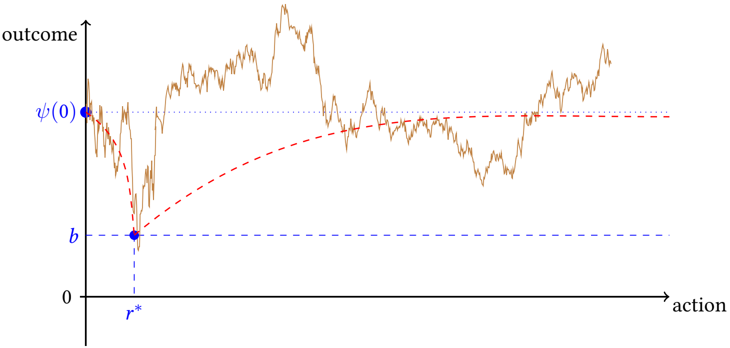
- For each $a \in \mathcal{A} = \mathbb{Z}$, there are two states ψ and ψ':
  - $\psi(a) = b$, $\psi(a+1) = 0$ and $\psi(a') = 100b \ \forall a' \in \mathcal{A} \setminus \{a, a+1\}$.
  - $\psi'(a) = b$, $\psi'(a+2) = 0$ and $\psi'(a') = 100b \ \forall a' \in \mathcal{A} \setminus \{a, a+2\}$.

Misalignment Without Directional Uncertainty



- For each $a \in \mathcal{A} = \mathbb{Z}$, there are two states ψ and ψ':
  - $\psi(a) = b$, $\psi(a+1) = 0$ and $\psi(a') = 100b \ \forall a' \in \mathcal{A} \setminus \{a, a+1\}$.
  - $\psi'(a) = b$, $\psi'(a+2) = 0$ and $\psi'(a') = 100b \ \forall a' \in \mathcal{A} \setminus \{a, a+2\}$.
- Efficient equilibrium exists if neither states dominate for any action.
- Receiver is never aligned with the sender *and* has no directional uncertainty.

Orstein-Uhlenbeck: Mean-Reversion



- The mapping is Ornstein-Uhlenbeck mean-reverting to $\psi(0)$.
- Expected outcome always points toward $\psi(0)$.
- First-point equilibrium exists $\forall b \in [0, \psi(0))$ and $\forall q \in \mathbb{R}$.

Details of OU process

Wiener State Space: Mean Reversion

Back to Extensions

- $\psi(a)$ is the solution to the stochastic differential equation:

$$d\psi(a) = -\kappa\left(\psi(0) - \psi(a)\right) da + \sigma\, dW(a)$$

- κ is the mean-reversion coefficient, and σ is the volatility term.
- Environment has the same state space as the Brownian environment.
- Differs in how the states are translated into outcomes via the outcome mappings.
- Deviations to $a < r^*$ are worse for the receiver by the continuity of OU process.
- For deviations $a > r^*$:

$$\mathbb{E}[\psi(a) \mid m^*(\psi) = r^*] = \underbrace{\psi(0) - (\psi(0) - \psi(r^*))\underbrace{\exp(-\kappa(a - r^*))}_{<1}}_{>\psi(r^*)}$$

$$\mathrm{Var}(\psi(a) \mid m^*(\psi) = r^*) = \frac{\sigma^2}{2\kappa}\left(1 - \exp[-2\kappa(a - r^*)]\right)$$

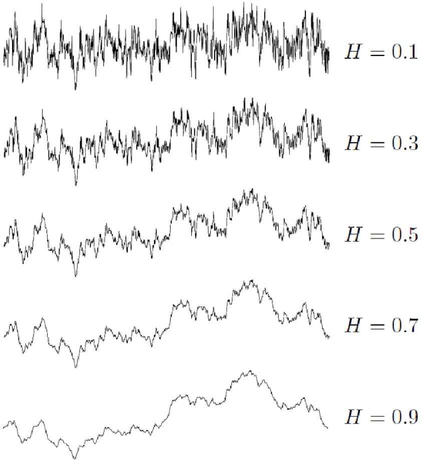
Wiener State Space: Non-Markovian

Back

- We can think of fractional BM as keeping the drift same and redefining the Cov $(\psi(a), \psi(a'))$ by:

$$\sigma^2 \frac{1}{2}\left(|a|^{2H} + |a'|^{2H} - |a - a'|^{2H}\right)$$

- H is the Hurst index describes the raggedness of the resultant motion:
  - If $H = 0.5$ then the state is Wiener process;
  - If $H > 0.5$ then the increments of the process are positively correlated;
  - If $H < 0.5$ then the increments of the process are negatively correlated.
- H changes the shape of the variance: Linear, Convex or Concave.



Wiener State Space: Non-Gaussian

Back to Extensions

- $\psi(a)$ is geometric Brownian Motion, which is the solution to the differential equation:

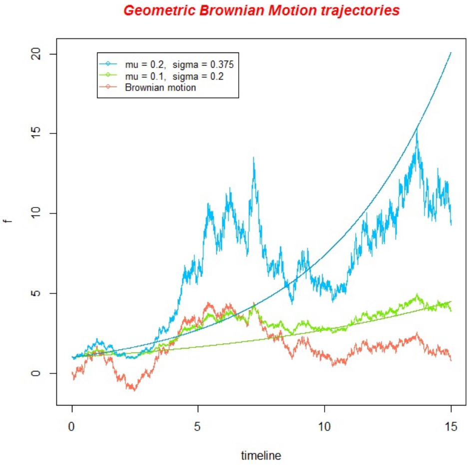
$$d\psi(a) = \mu\psi(a)dt + \sigma\psi(a)dW(a).$$

- The solution is given by:

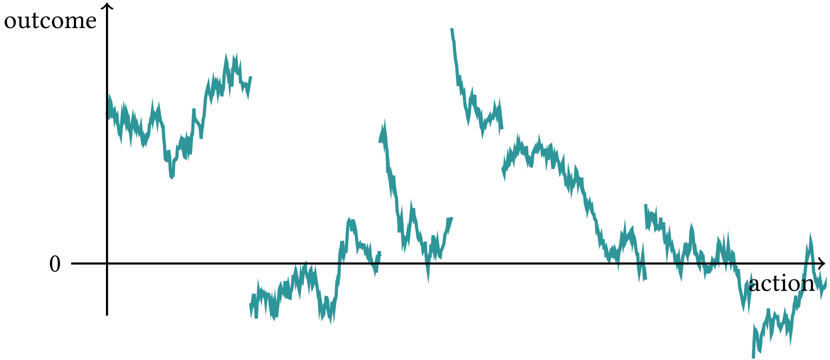
$$\psi(a) = \psi_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W(a)\right).$$

- $\psi(a)$ is log-normally distributed with:

$$\mathbb{E}[\psi(a)] = [\psi_0 \exp(\mu a)]$$
$$\mathrm{Var}(\psi(a)) = \psi_0^2 \exp(2\mu a)\left(\exp(\sigma^2 a) - 1\right).$$



Wiener State Space: Discontinuous



- $\psi(a)$ = Wiener process $W(a)$ + compound Poisson process $Y(a)$:

$$\psi(a) = \mu t + \sigma W(a) + Y(a)$$

- If $Y(a) \geq 0$, then our techniques based on first hitting times directly apply.

Back to Extensions

Wiener State Space: Higher Dimensions

Back to Extensions

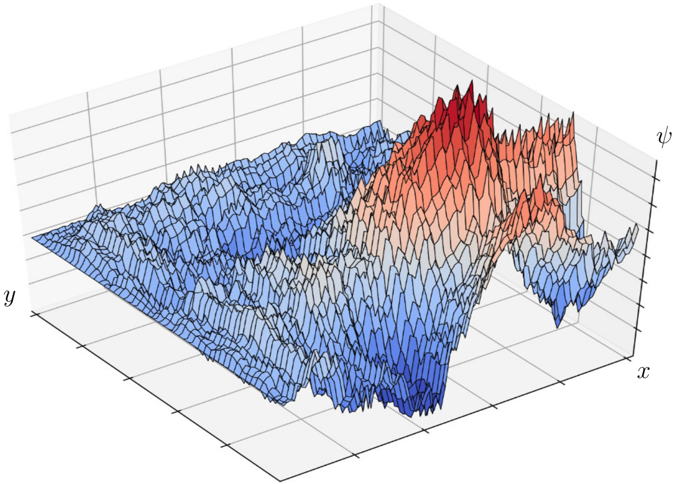
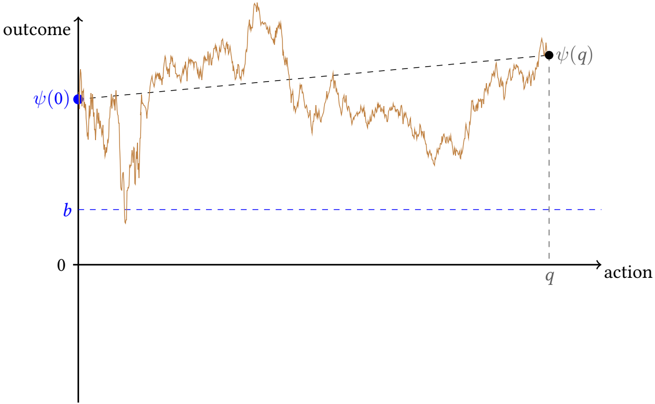


Figure: Brownian Sheet $\psi : X \times Y \to \mathbb{R}$.

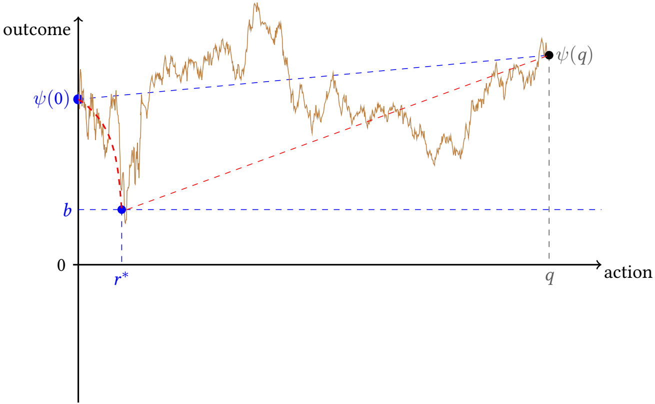
Wiener State Space: More Knowledge



- Consider the Brownian Motion environment.
- But, the receiver begins knowing a second point action q where $\psi(q) \geq \psi(0)$.

Back to Extensions

Wiener State Space: More Knowledge



Back to Extensions

- Consider the Brownian Motion environment.
- But, the receiver begins knowing a second point action q where $\psi(q) \geq \psi(0)$.

- Similar to the OU process
- Easy to satisfy the first-point equilibrium.